Universal Control of an Oscillator with Dispersive Coupling to a Qubit

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Ancient History and Recent Advances

In 1995 Law and Eberly devised an efficient protocol for the preparation of an arbitrary state in an oscillator-qubit system governed by the Jaynes-Cummings Hamiltonian [1]. However the more complex problem of implementing arbitrary unitary operations remains an outstanding challenge. Even with recent advances [2][3][4], protocols suffer from various inefficiencies and lack experimental implementations. Meanwhile, development of superconducting circuits in the strong dispersive regime opens new possibilities for efficient universal control of the oscillator that we exploit in the present protocol.

What is Universal Quantum Control?

Having universal control of the system implies the ability to perform any unitary operation on it without the need to know its initial state (i.e. apply any unitary matrix to the state of the system). It is a much more general capability than the usual state preparation routines where we constrain only one column of the unitary matrix (taking the ground state to the target state).

Dispersively Coupled Qubit

The Hamiltonian of the dispersively coupled qubit-oscillator system is

$$\hat{H} = (\omega_q - \chi\hat{n}) \mid e\rangle\langle e \mid + \hat{H}_1 + \hat{H}_2, \qquad (1)$$

with time-dependent drive of the oscillator

$$\hat{H}_1 = \epsilon(t)\,\hat{a}^\dagger + h.c., \qquad (2)$$

and time-dependent drive of the qubit

$$\hat{H}_2 = \Omega(t)\,e^{i\omega_q t}\,|e\rangle\langle g| + h.c., \qquad (3)$$

where ω_q is the qubit frequency; $\hat{a}^\dagger$ and $\hat{a}$ are the raising and lowering operators; $\hat{n} = \hat{a}^\dagger\hat{a}$; χ is the dispersive coupling; $\Omega(t)$ and $\epsilon(t)$ are the time-dependent drives of the qubit and the oscillator.

Basic "Buildingblock" Operations Available in this System The SNAP Gate

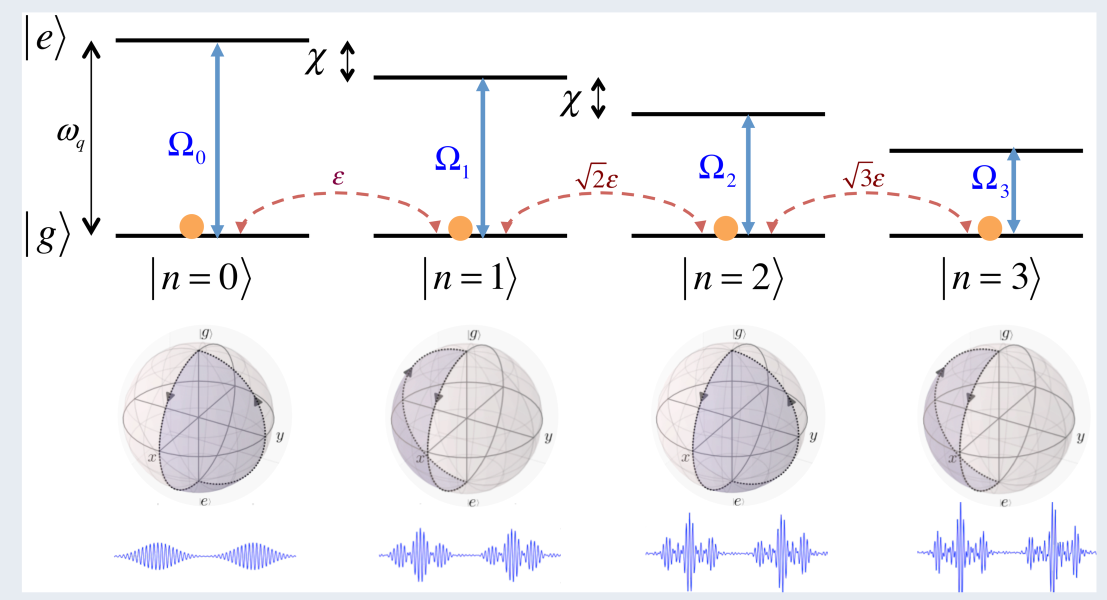

Figure 1: Energy level diagram of the qubit-oscillator system in the rotating frame of the oscillator. A weak displacement operation (red dashed arrows) couples the states $|g, n-1\rangle$ and $|g, n\rangle$ for all n. The SNAP gate (blue solid arrows) can simultaneously accumulate different Berry phases $\{\theta_n\}$ to states $\{|g, n\rangle\}$. The Berry phase θ_n is proportional to the enclosed shaded area in the corresponding Bloch sphere, achieved by resonant microwave pulses with frequency $\omega_q - n\chi$ (blue traces).

We will control only the ground subspace of the system $\{|g, n\rangle\}_n$ and use the excited subspace $\{|e, n\rangle\}_n$ as an auxiliary and do not keep any population in it between operations.

$\hat{H}_1$ provides us with a displacement operation:

$$\hat{D}(\alpha) = \exp\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right). \qquad (4)$$

$\hat{H}_2$ provides us with *selective on number of photons* Rabi oscillations if $|\Omega(t)| \ll \chi$. Each of the two-level subsystems can move on an arbitrary path on the Bloch sphere. On return to the ground state each subsystem will acquire a Berry phase. The resulting operation is the SNAP gate (*Selective on Number Arbitrary Phase gate*):

$$\hat{S}(\vec{\theta}) = \sum_{n=0}^{\infty} e^{i\theta_n} \mid n\rangle\langle n \mid, \qquad (5)$$

where $\vec{\theta} = \{\theta_n\}_{n=0}^{\infty}$ is the list of phases.

Implementing a Rotation between Two Neighboring States

The operation

$$\hat{V}_n = \hat{D}_{\alpha_1^{(n)}}\,\hat{R}_n\,\hat{D}_{\alpha_2^{(n)}}\,\hat{R}_n\,\hat{D}_{\alpha_3^{(n)}} \approx e^{[-i\theta(|n\rangle\langle n+1| + |n+1\rangle\langle n|)]}$$

where $\hat{R}_n = -\sum_{n'=0}^{n} |n'\rangle\langle n'| + \sum_{n'=n+1}^{\infty} |n'\rangle\langle n'|$

performs a high fidelity rotation of a fixed angle in $\{|n\rangle, |n+1\rangle\}$. $\hat{R}_n$ is a SNAP gate that changes the sign of the first n states and commutes with $\hat{D}_\alpha$ over all but the $\{|n\rangle, |n+1\rangle\}$ subspace.

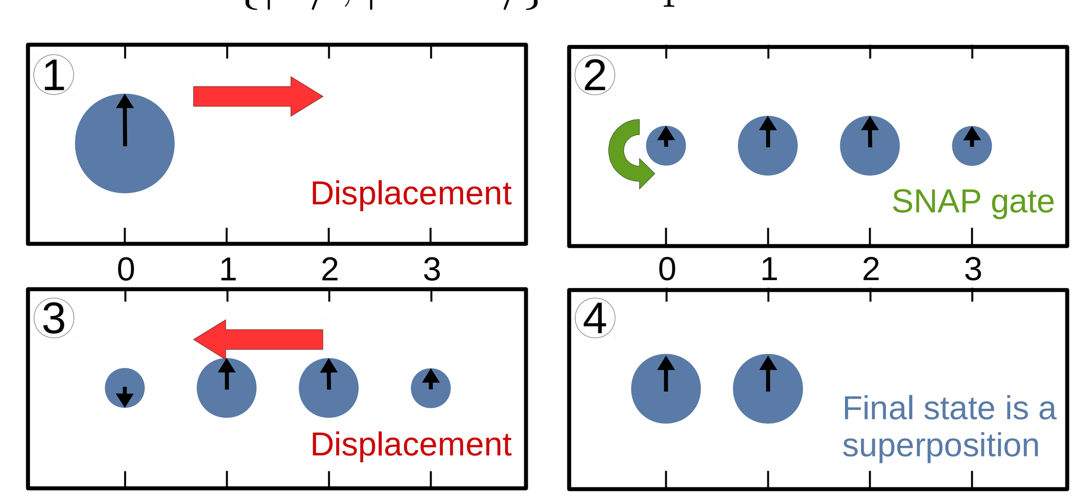

Figure 2: Pictorial representation of the steps implementing a rotation between $|0\rangle$ and $|1\rangle$. The horizontal axis enumerates number states. The area of the circles is the population in the given state. The arrow is the phase.

Implementing an Arbitrary Unitary Operation

Having $SO(2)$ rotation operations (like $\hat{V}_n$) and $U(1)$ phase inducing operations (like $\hat{S}(\vec{\theta})$) we can chain them in any number of ways and efficiently build any unitary operation.

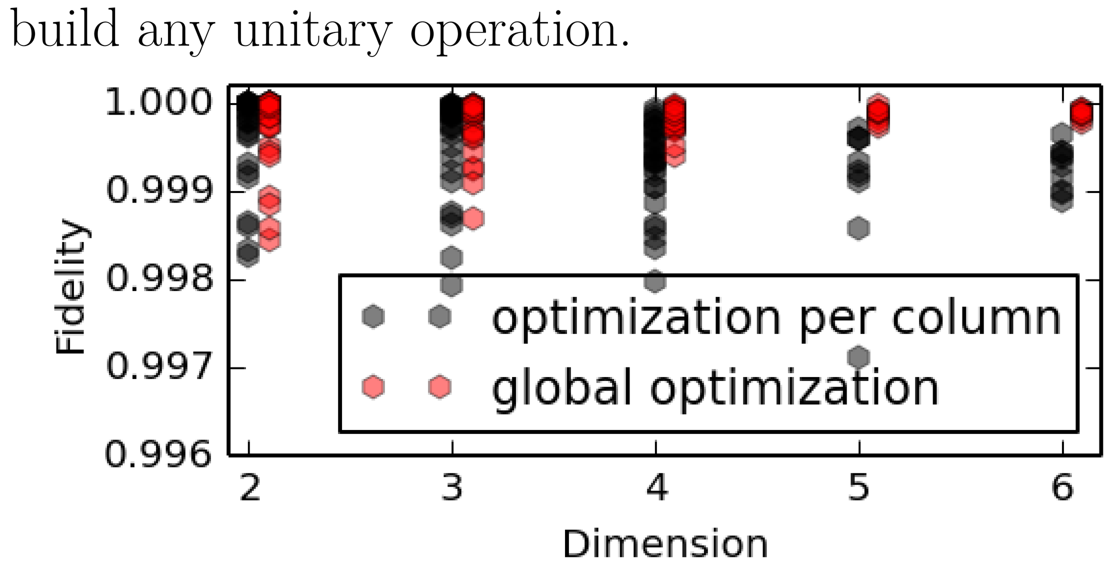

Figure 3: To test the entire protocol, we randomly selected "target" unitary operations from the $U(N)$ group (N from 2 to 6) and let our algorithm design control pulses that implement them with high fidelity $F = \frac{1}{N_c}\left|Tr\left(\hat{U}^\dagger_{\text{construct}}\hat{U}_{\text{target}}\right)\right|$. Each point represents one run of our algorithm trying to implement a randomly selected "target" unitary (two colors for two different optimization strategies).

Conclusions and Outlook

The dispersive Hamiltonian permits selective control (the SNAP gate) which in turn leads to our protocol for universal control, which is both:

- efficient, requiring only $\mathcal{O}(N^2/\chi)$ time to perform an $N \times N$ unitary operation;
- high fidelity, performing consistently at fidelities above 0.999 and permitting efficient fidelity-time tradeoffs in case higher fidelities are required. For target infidelity ε it requires only $\mathcal{O}(N^3/\sqrt{\varepsilon})$ steps.

In addition:

- For 'sparse' matrices or sparse 'states' the protocol can be further optimized by skipping unnecessary operations. For instance we can prepare a number state $|n\rangle$ in only $\mathcal{O}(\sqrt{n})$ operations instead of the usual $\mathcal{O}(n)$.
- The protocol can be generalized to perform a unitary operation on the entire Hilbert space instead of being restricted to working on only the $\{|g, n\rangle\}_n$ subspace.

References

For the details of our work:

- Krastanov et al., *Universal Control of an Oscillator with Dispersive Coupling to a Qubit* (PRA 92, 040303, 2015);
- Experimental implementation: Heeres et al., *Cavity State Manipulation Using Photon-Number Selective Phase Gates* (PRL 115, 137002, 2015)

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