UNIVERSAL CONTROL OF AN OSCILLATOR WITH DISPERSIVE COUPLING TO A QUBIT

arXiv preprint 1502.08015

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OUTLINE

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- A New Phase-rotation Gate for a Cavity-Qubit System
- Performing Arbitrary Unitary Operations on the Cavity

- For an experimentalists view:
  - **Y39.00005 (by Reinier Heeres in 30 minutes) and arXiv 1503.01496**
- See also:
  - (theory) **Law & Eberly (PRL 76, 1055)**
  - (theory) **Mischuck & Mølmer (PRA 87, 022341)**
  - (theory) **Santos (PRL 95, 010504)**
  - (experiment) **Leibfried et al. (Rev Mod Phys 75, 281)**
  - (experiment) **Hofheinz et al. (Nature 459, 546)**

THE SYSTEM AND THE DRIVES

THE SYSTEM

Cavity (oscillator) coupled to a qubit (a two-level system)

$$\hat{H}_0 = \omega_q \mid e\rangle\langle e \mid + \omega_c \hat{n} - \chi \mid e\rangle\langle e \mid \hat{n}$$

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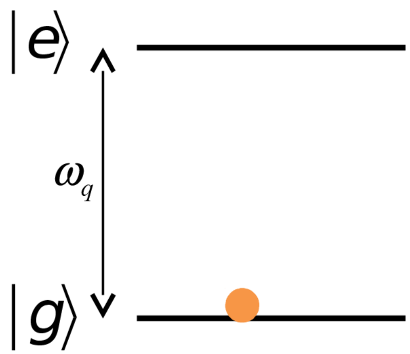

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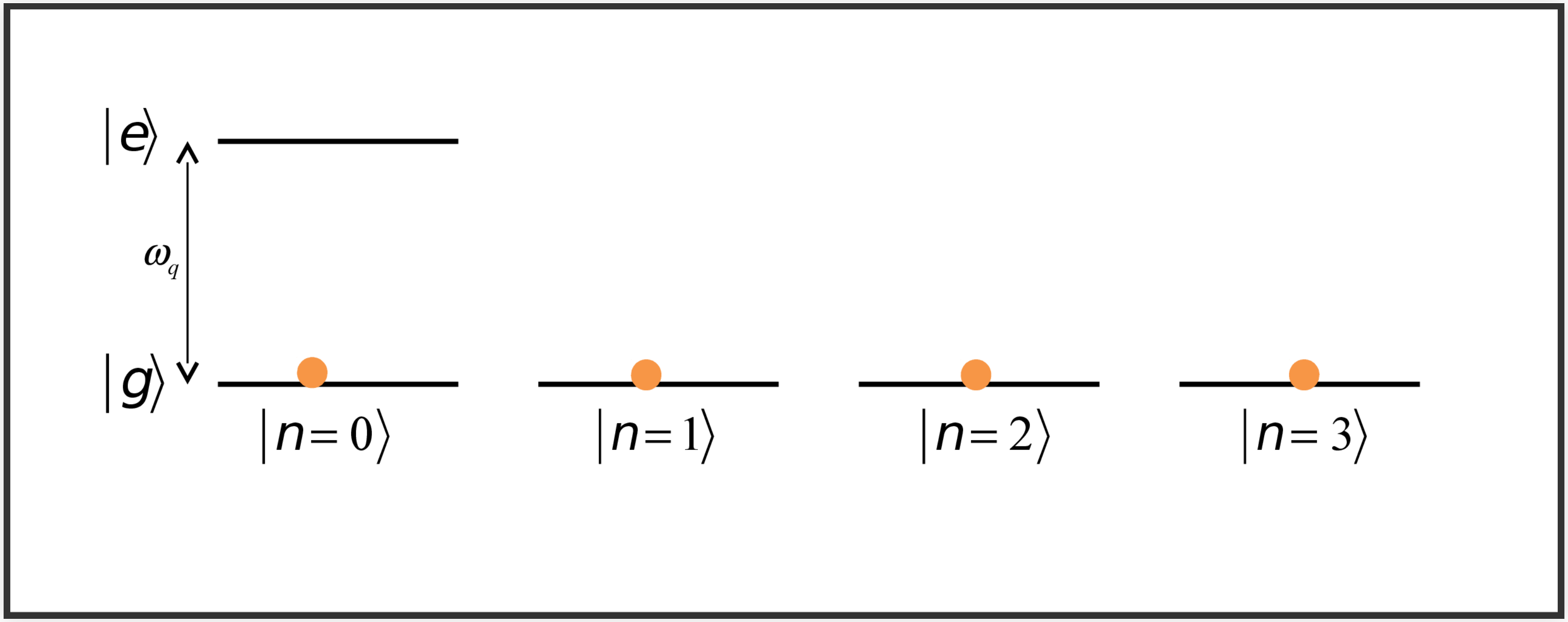

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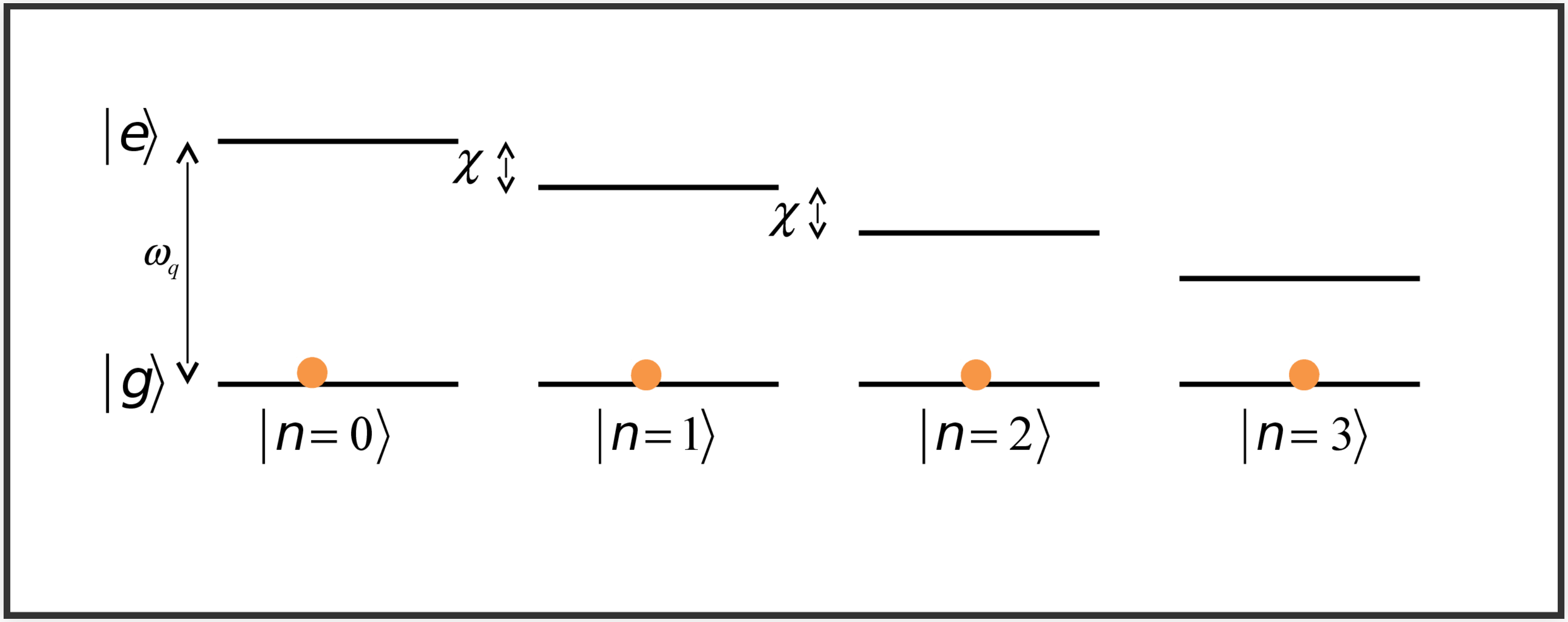
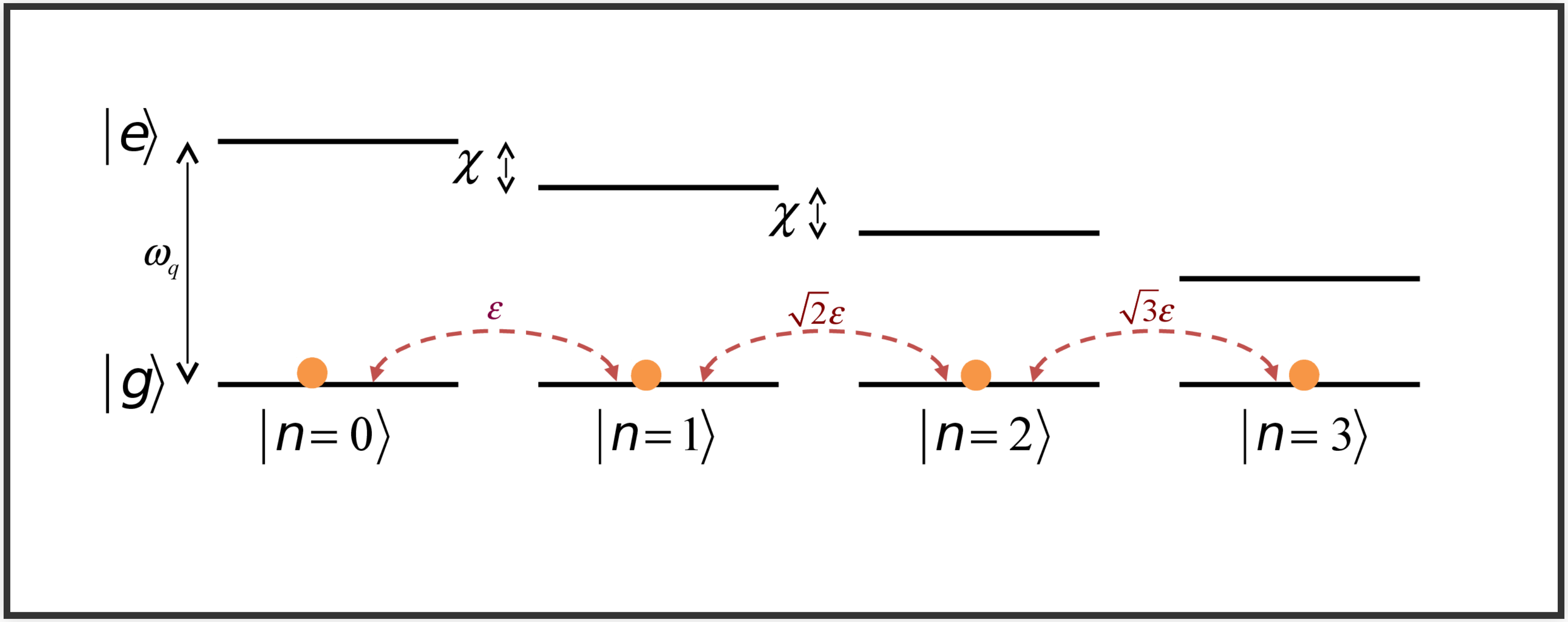


THE DRIVES - CONTROLLING THE CAVITY

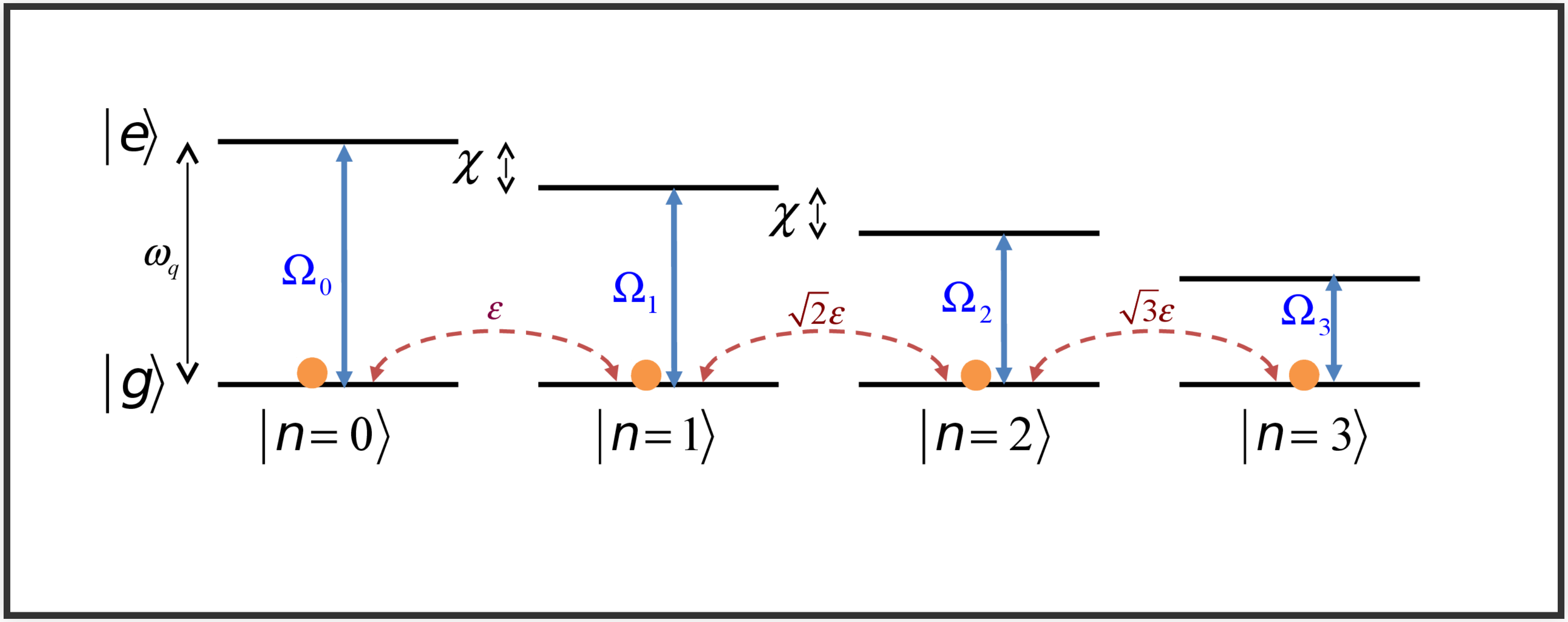
$$\hat{H}_{cavity} = \varepsilon\left(t\right) e^{i\omega_c t}\hat{a}^{\dagger} + h.c.$$



THE DRIVES - CONTROLLING THE QUBIT

$$\hat{H}_{qubit} = \Omega\left(t\right) e^{i\omega_q t} \left|e\right\rangle\left\langle g\right| + h.c.$$

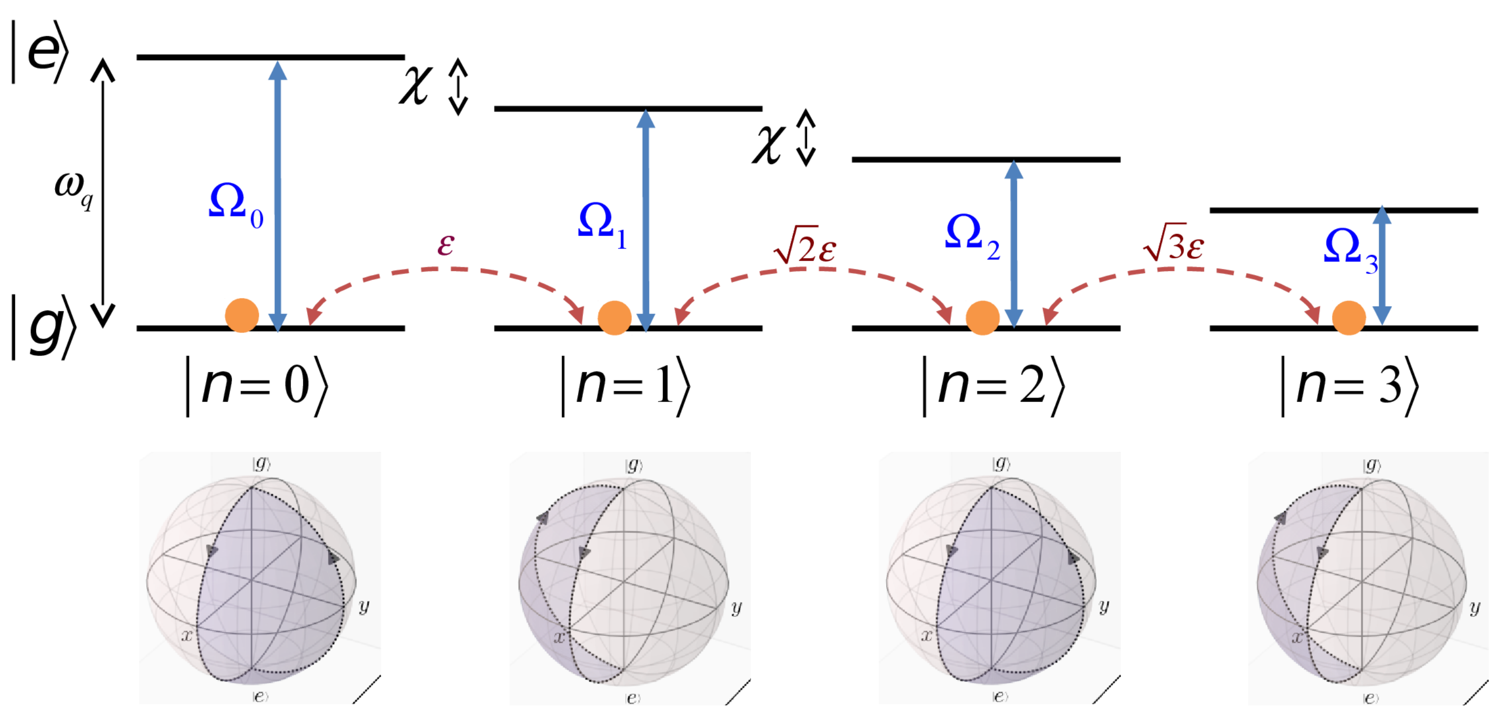
Selective on the number of photons: $\Omega(t) = \Omega e^{-in\chi t}$ with $\Omega \ll \chi$



'SNAP' GATE

Selective on **N**umber **A**rbitrary **P**hase $|g, n\rangle \rightarrow e^{i\theta_n}|g, n\rangle$

$$\hat{S}_n(\theta_n) = e^{i\theta_n |n\rangle\langle n|}$$



'SNAP' GATE

Selective on **N**umber **A**rbitrary **P**hase

On one two-level subsystem:

- $\hat{S}_n\left(\theta_n\right) = e^{i\theta_n|n\rangle\langle n|}$

On all two-level subsystems at once:

- $\hat{S}\left(\vec{\theta}\right) = \sum_{n=0}^{\infty} e^{i\theta_n} \mid n\rangle\langle n|$

UNIVERSAL CONTROL BY COMBINING SNAPS AND DISPLACEMENTS

PROOF OF UNIVERSALITY

- Displacement: $\hat{D}(\alpha) = \exp\left(\alpha \hat{a}^\dagger - \alpha^* \hat{a}\right)$
- SNAP Gate: $\hat{S}\left(\vec{\theta}\right) = \sum_{n=0}^{\infty} e^{i\theta_n} \mid n\rangle\langle n|$

The group commutator of SNAP gates and Displacements can couple any neighboring pair of number states:

$$\hat{D}(\epsilon)\hat{S}(\vec{\theta}_\epsilon)\hat{D}(-\epsilon)\hat{S}(-\vec{\theta}_\epsilon)$$

$$\approx \exp\left(i\epsilon^2\sqrt{n+1} \mid n\rangle\langle n+1 \mid +h.c.\right)$$

for some fixed n and an appropriate SNAP gate.

- See also:
  - **Lloyd & Braunstein (PRL 82, 1784)**

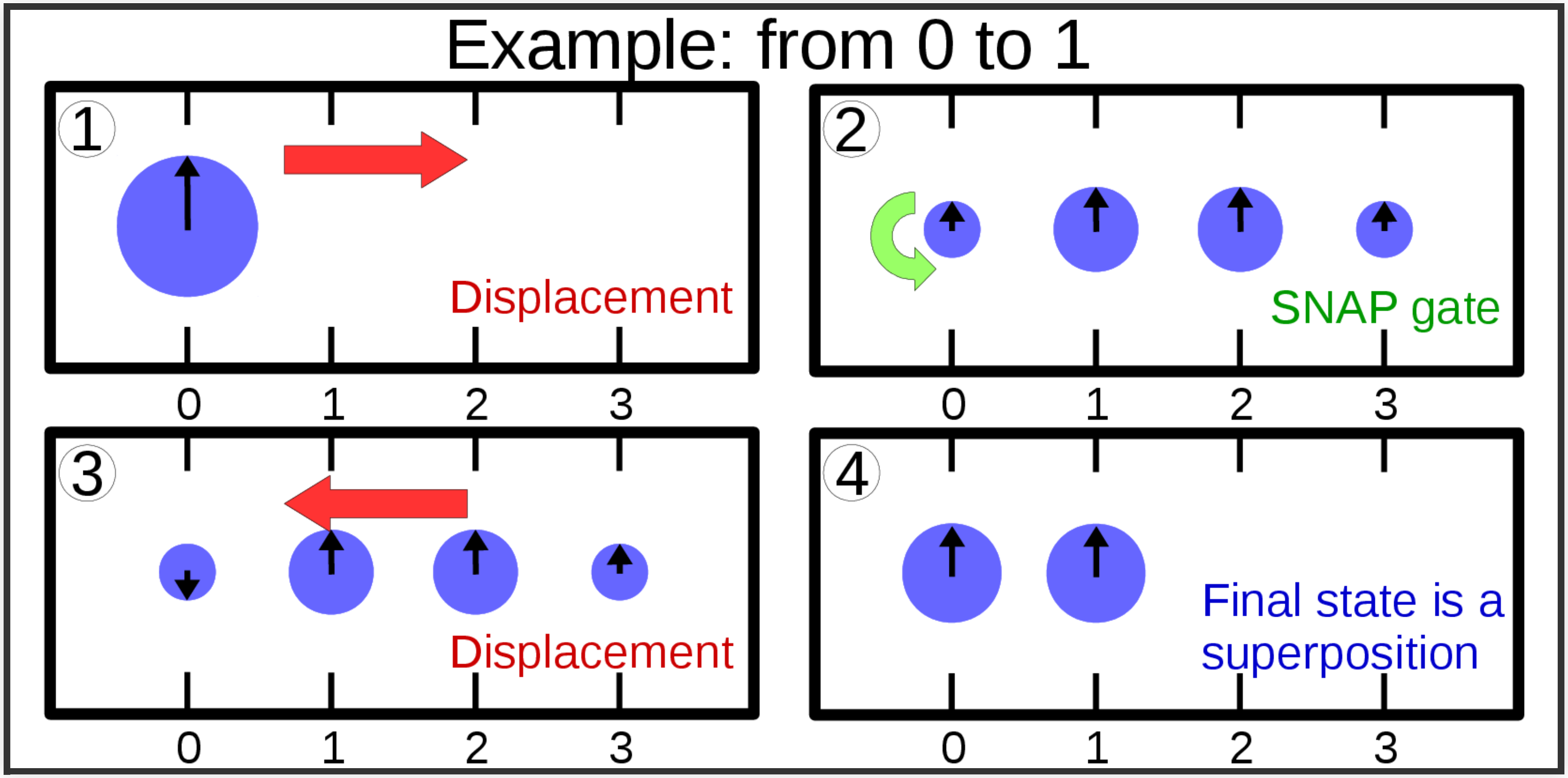
EXPLICIT ALGORITHM - N TO N+1

How to perform $|n\rangle \rightarrow \cos(\theta)\,|n\rangle + \sin(\theta)\,|n+1\rangle$?

EXPLICIT ALGORITHM - N TO N+1

$$\hat{U}_n = \hat{D}(\alpha_1)\hat{R}_n\hat{D}(\alpha_2)\hat{R}_n\hat{D}(\alpha_3)$$

$$\hat{R}_n = -\sum_{n'=0}^{n} |n'\rangle\langle n'| + \sum_{n'=n+1}^{\infty} |n'\rangle\langle n'|$$



UNIVERSAL CONTROL

Use the 2D rotations to prepare each column of the matrix one by one.

N-BY-N UNITARY MATRIX

We want to construct $\hat{U}_{target}$

$$\hat{U}_{target}^{-1} = \left[\begin{array}{c|c} \hat{W}_{n\times n} & 0 \\ \hline 0 & Id \end{array}\right]$$

N-BY-N UNITARY MATRIX - REMOVING A COLUMN

Chain $N-1$ 2-dimensional rotations:

$$\hat{V}_{n-1,n}\dots\hat{V}_{1,2}\hat{S}_n\hat{U}_{target}^{-1}=\left[\begin{array}{c|c}\begin{array}{c|c}\hat{W}_{n-1\times n-1} & 0\\\hline 0 & 1\end{array} & 0\\\hline 0 & Id\end{array}\right]$$

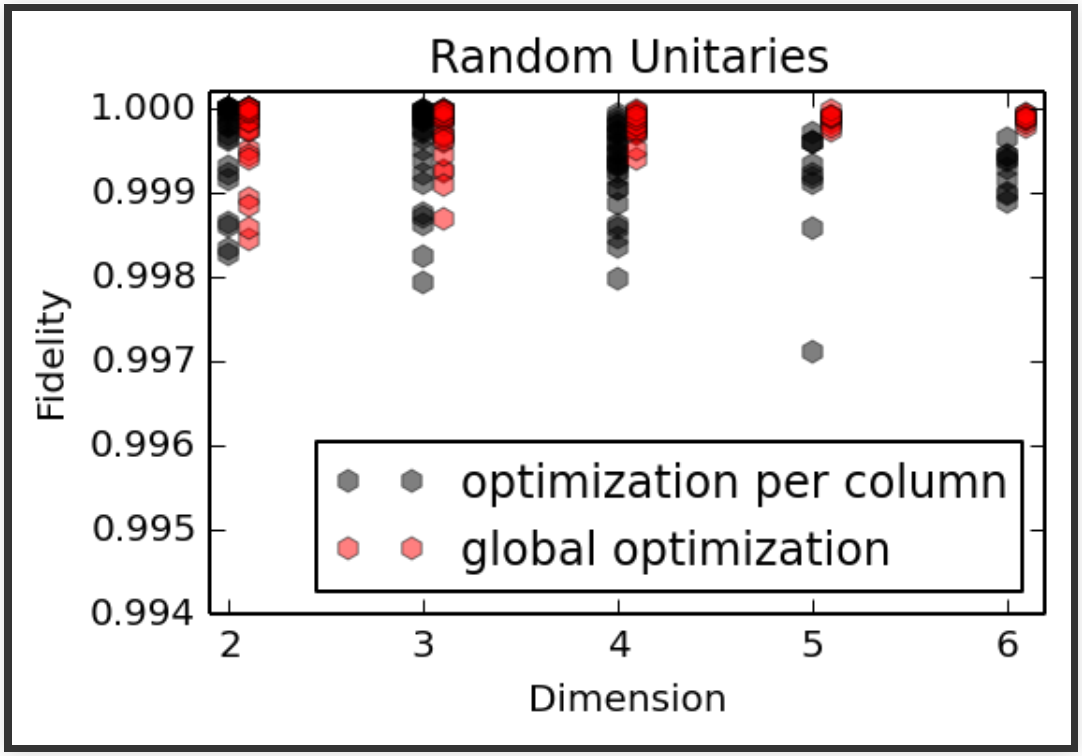
N-BY-N UNITARY MATRIX

We repeat the procedure for all columns optimizing the fidelity

$$F = \left| \frac{1}{N_{cutoff}} Tr\left(\hat{U}^\dagger \hat{U}_{target} \right) \right|$$

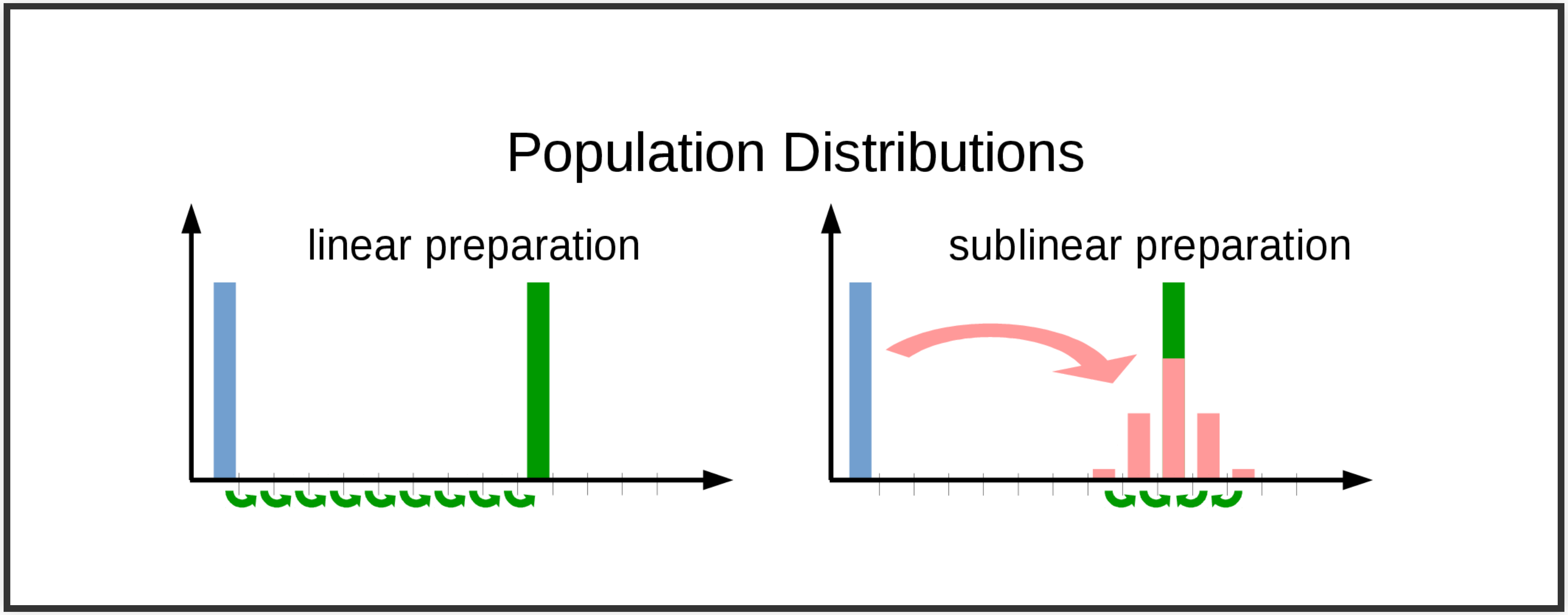
N-BY-N UNITARY MATRIX

Fidelity for a random sample of unitary matrices:



SUPER EFFICIENT FOCK STATE PREPARATION

- Displace to coherent state $|\alpha\rangle = D(\alpha = \sqrt{n}) \mid 0\rangle$
- Use $\mathcal{O}(\sqrt{n})$ rotations to "fold" it into $|n\rangle$

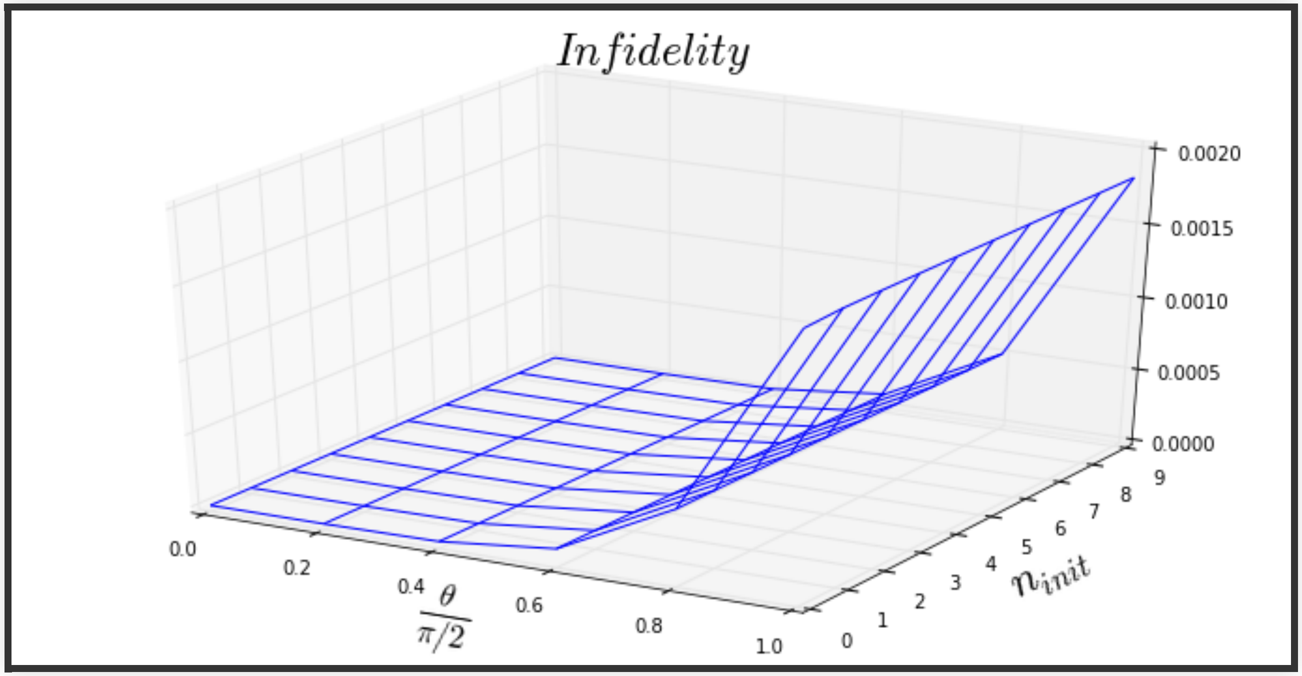


TALLY THE COST

- To perform an arbitrary N-by-N unitary matrix
  - $\mathcal{O}(N^2)$ gates neccessary
- To prepare a state with an N-dimensional cutoff:
  - $\mathcal{O}(N)$ gates neccessary
  - $\mathcal{O}(\sqrt{N})$ gates neccessary for "sparse" states

ASYMPTOTIC FIDELITY

Consider again 2D rotations and look at small θ:



ASYMPTOTIC FIDELITY

We can prove

$$\#\text{gates} \propto \frac{N^3}{\sqrt{\text{infid}}}$$

$$\text{Time} \propto \frac{N^3}{\sqrt{\text{infid}}}\ \frac{1}{\chi}$$

Compare to **Law & Eberly (PRL 76, 1055)**: Only State Preparation

Compare to **Mischuck & Mølmer (PRA 87, 022341)**: Universal Control at $\text{time} \propto \frac{N^{18.5}}{\text{infid}^3}\ \frac{1}{g}$

CONCLUSION

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arXiv preprint **1502.08015** (and experiment at **1503.01496**)

- Efficient State Preparation
  - Including an Optimization for Fock States
- Efficient Universal Control
  - Satisfactory at "first-pass" level
  - Remains Efficient in the Asymptotic Regime
- People are Already Implementing it in Schoelkopf's group at Yale
  - Y39.00005 (by Reinier Heeres in 20 minutes)