UNIVERSAL CONTROL OF AN OSCILLATOR WITH DISPERSIVE COUPLING TO A QUBIT

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Acknowledgement: Michel Devoret, Steven Girvin, Zaki Leghtas, Mazyar Mirrahimi, Andrei Petrenko, Matt Reagor, and many others from the YQI.

MLS, FEBRUARY 2, 2015

OSCILLATORS ARE NICE

Especially when coupled to a spin.

Even more so if they are the relatively long lived and fast cavities you guys are constructing on the 4th floor.

OUTLINE

OUTLINE

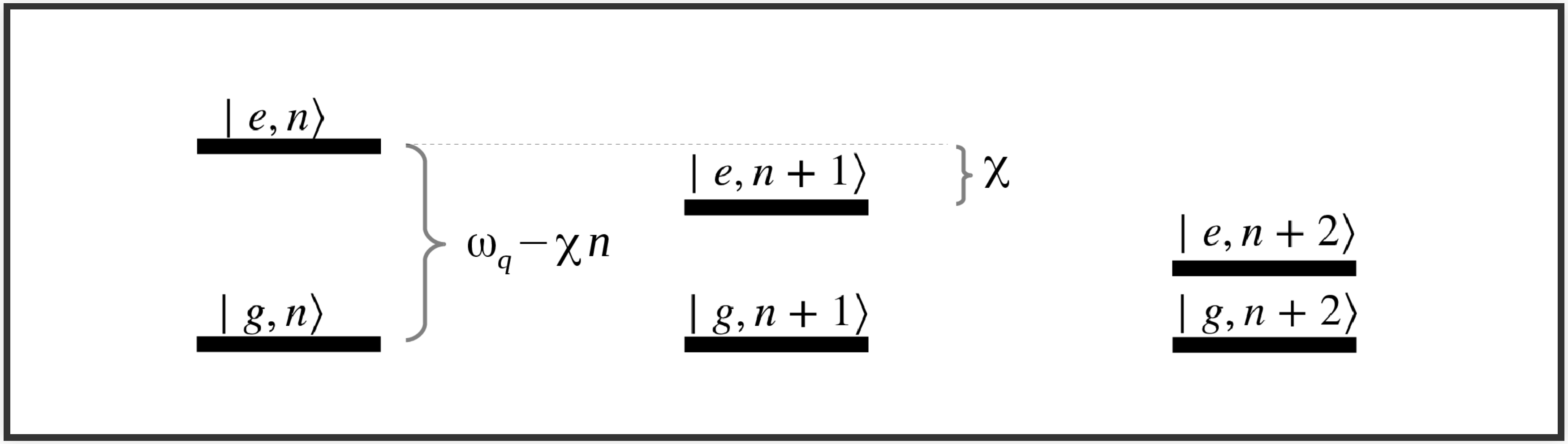
- The System
- SNAP Gate
- State Preparation
- Universal Control
- Summary and Outlook

THE SYSTEM AND THE DRIVES

THE SYSTEM

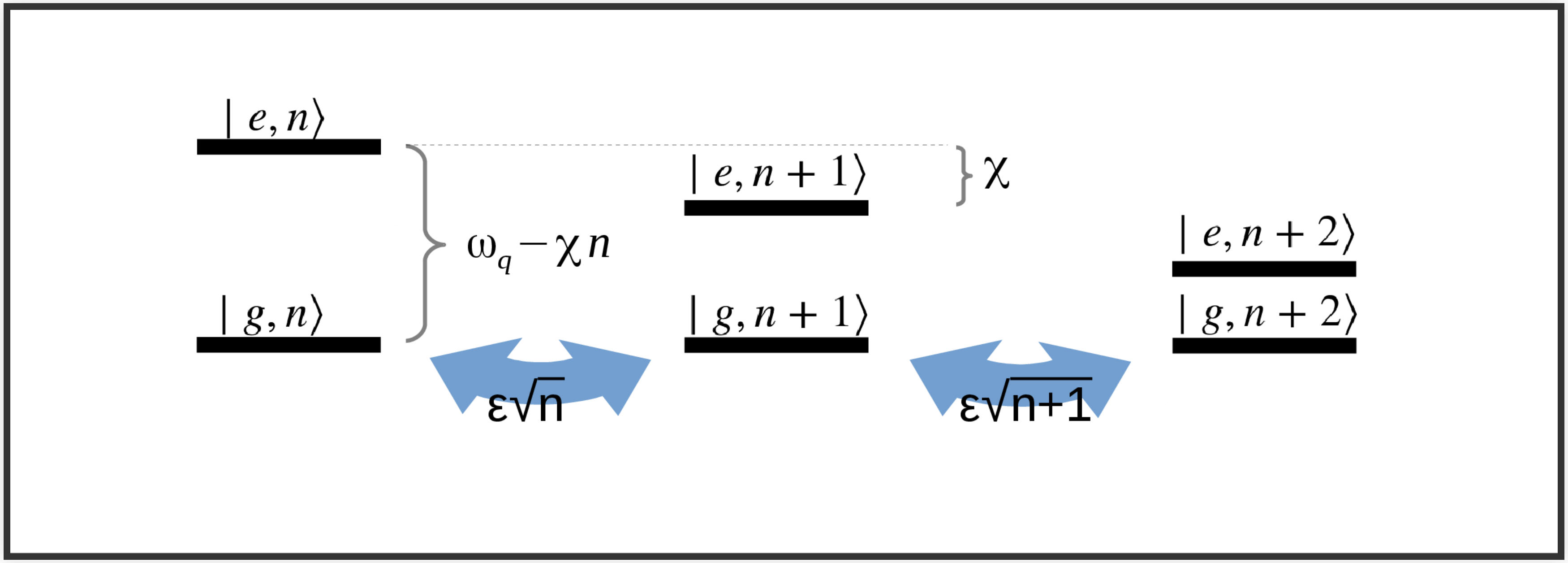
Cavity (EM oscillator) coupled to a qubit (a two-level system)

$$\hat{H}_0 = \omega_q \mid e\rangle\langle e \mid + \omega_c \hat{n} - \chi \mid e\rangle\langle e \mid \hat{n}$$



THE DRIVES - CONTROLLING THE CAVITY

$$\hat{H}_{cavity} = \varepsilon\left(t\right) e^{i\omega_c t}\hat{a}^\dagger + h.c.$$

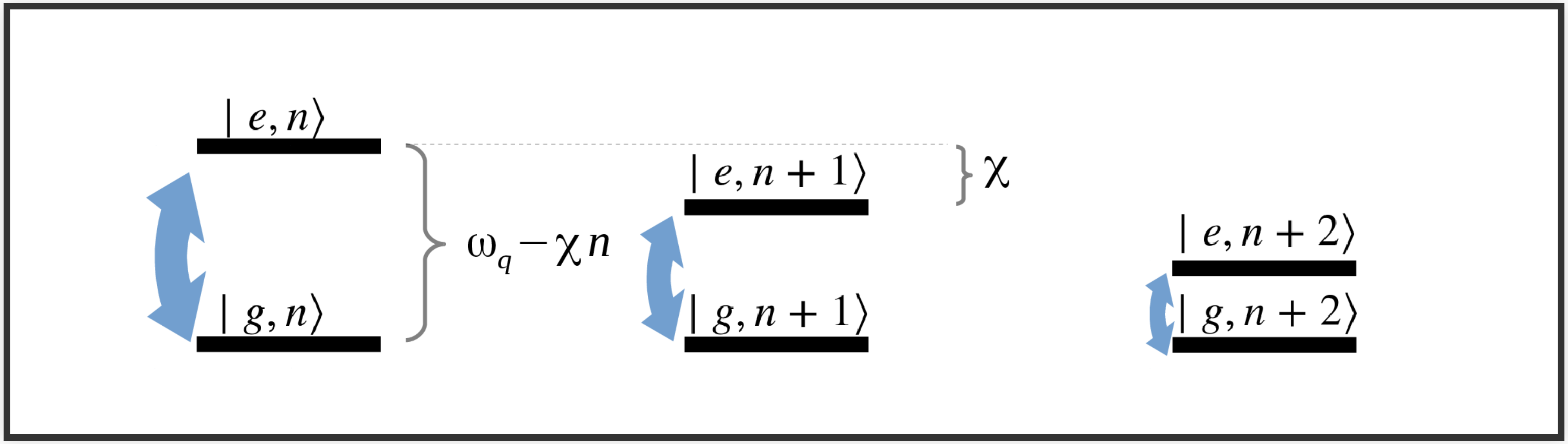


Displacement operator: $\hat{D}(\alpha) = \exp\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)$ where $\alpha = i\int\varepsilon(t)dt$ acting only on the ground subspace

$$\{|0\rangle\ldots|n\rangle\ldots\}\otimes|g\rangle$$

THE DRIVES - CONTROLLING THE QUBIT

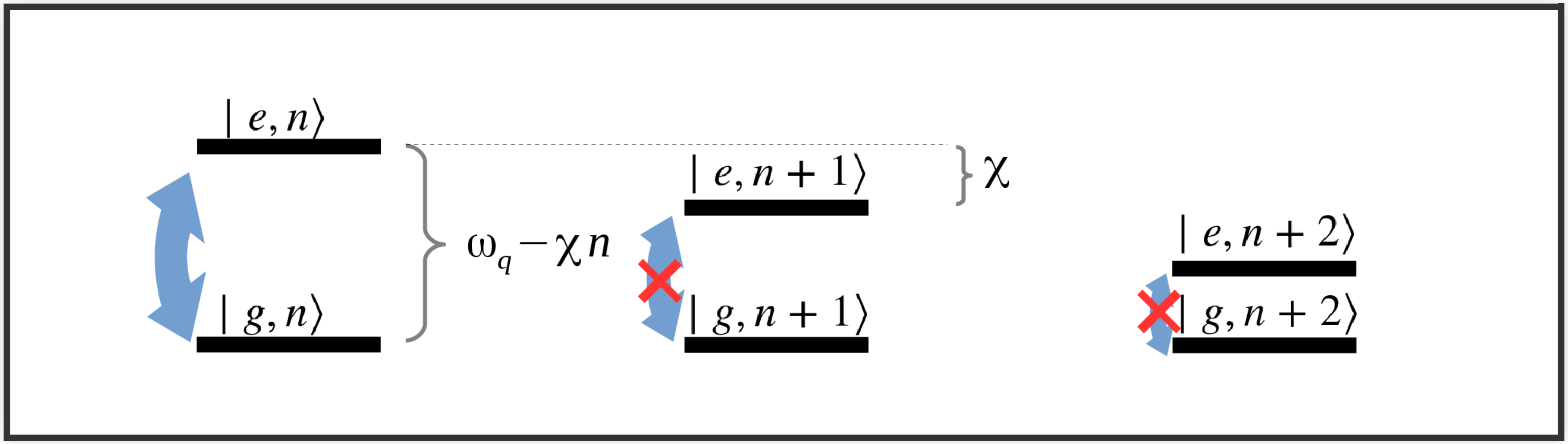
$$\hat{H}_{qubit} = \Omega\left(t\right) e^{i\omega_q t} \left|e\right\rangle\left\langle g\right| + h.\,c.$$



THE DRIVES - CONTROLLING THE QUBIT

The control can be selective on the number of photons!

$$\Omega(t) = \Omega e^{-in\chi t} \text{ with } \Omega \ll \chi$$



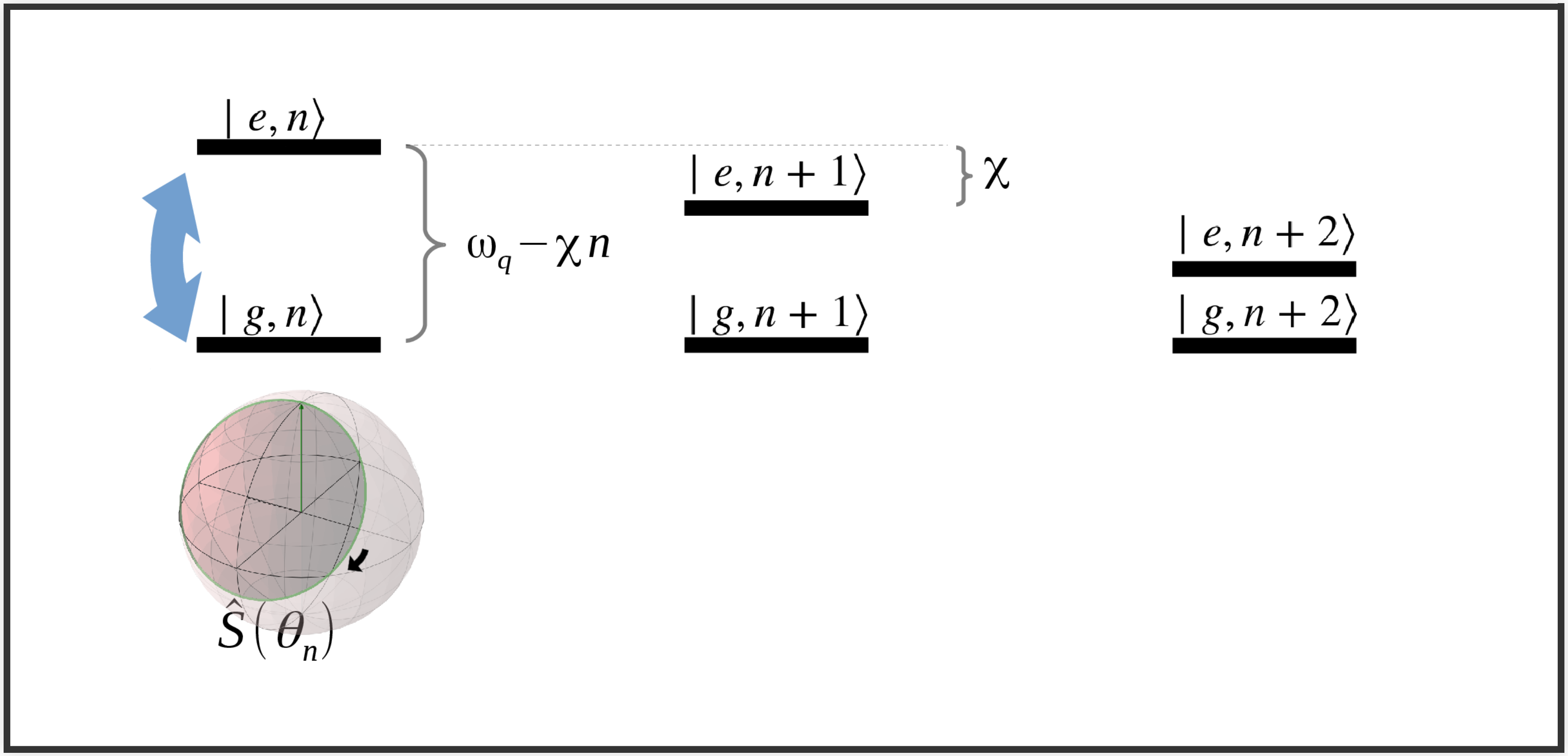
TOGETHER

$$H = H_0 + H_{cavity}(t) + H_{qubit}(t)$$

SNAP GATE

'SNAP' GATE

Use the selective control on the qubit to take closed paths on the Bloch sphere. **Always end in the ground state.**



$$|g,n\rangle \rightarrow e^{i\theta_n}\,|g,n\rangle$$

'SNAP' GATE

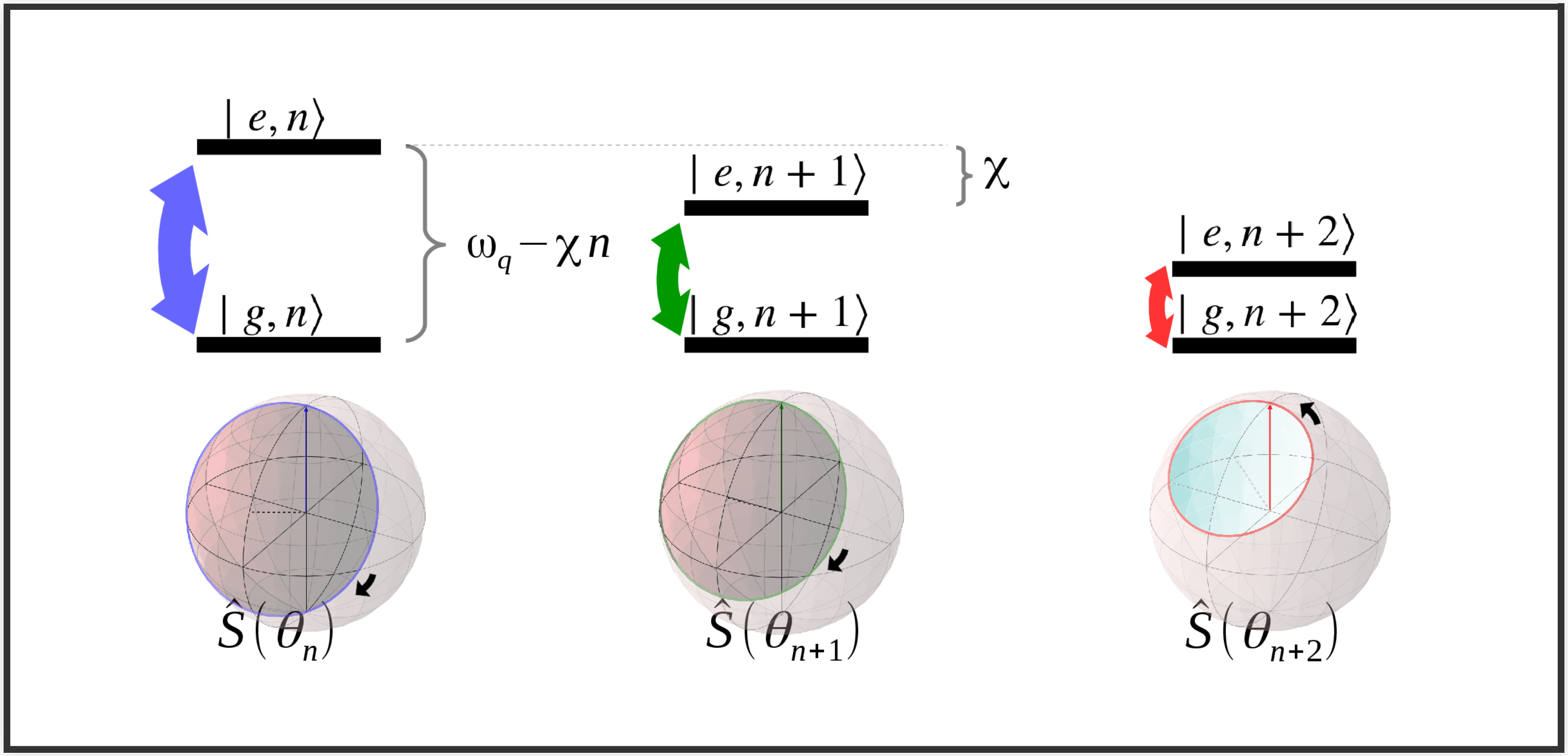
Selective on **N**umber **A**rbitrary **P**hase

$$\hat{S}_n\left(\theta_n\right) = e^{i\theta_n |n\rangle\langle n|}$$

PARALLEL 'SNAP' GATE

We can address multiple pairs of states in parallel.

$$\hat{S}\left(\vec{\theta}\right) = \prod_{n=0}^{\infty} \hat{S}_n\left(\theta_n\right) = \sum_{n=0}^{\infty} e^{i\theta_n} \mid n\rangle\langle n|$$



RESTRICTED TO THE GROUND STATE

The **usable** Hilbert space is the $\{|0\rangle \ldots |n\rangle \ldots\} \otimes |g\rangle$ subspace.

For most of the rest of the presentation we will restrict ourselves to the ground subspace.

STATE PREPARATION

STATE PREPARATION

We have these two basic operation acting on the ground subspace (processor instructions in a CPU analogy):

- Displacement:

$$\hat{D}(\alpha) = \exp\left(\alpha \hat{a}^\dagger - \alpha^* \hat{a}\right)$$

- SNAP Gate:

$$\hat{S}\left(\vec{\theta}\right) = \prod_{n=0}^{\infty} \hat{S}_n\left(\theta_n\right) = \sum_{n=0}^{\infty} e^{i\theta_n} \mid n\rangle\langle n|$$

Both of which act only on the ground subspace.

STATE PREPARATION

Can we use them to prepare any state in the ground subspace?

Consider $\vec{\theta}_\epsilon = (\underbrace{\epsilon, \ldots, \epsilon}_{n}, 0, \ldots)$

Sandwich the corresponding SNAP gate with a similar Displacement gate into a group commutator:

$$\hat{D}(\epsilon)\hat{S}(\vec{\theta}_\epsilon)\hat{D}(-\epsilon)\hat{S}(-\vec{\theta}_\epsilon)$$

$$\approx \exp\left(i\epsilon^2\sqrt{n+1}(|\, n\rangle\langle n+1 \,| + h.c.)\right)$$

Nearest neighbours are coupled and by iteration we can couple all levels.

EXPLICIT ALGORITHM - N TO N+1

Given the state $|n\rangle$ we want to create the state

$$|\text{target}\rangle = \cos(\theta)\,|n\rangle + \sin(\theta)\,|n+1\rangle$$

Inspired by the above, consider the non-infinitesimal operation:

$$\hat{U}_n = \hat{D}(\alpha_1)\hat{R}_n\hat{D}(\alpha_2)\hat{R}_n\hat{D}(\alpha_3)$$

where

$$\hat{R}_n = -\sum_{n'=0}^{n} |n'\rangle\langle n'| + \sum_{n'=n+1}^{\infty} |n'\rangle\langle n'|$$

EXPLICIT ALGORITHM - N TO N+1

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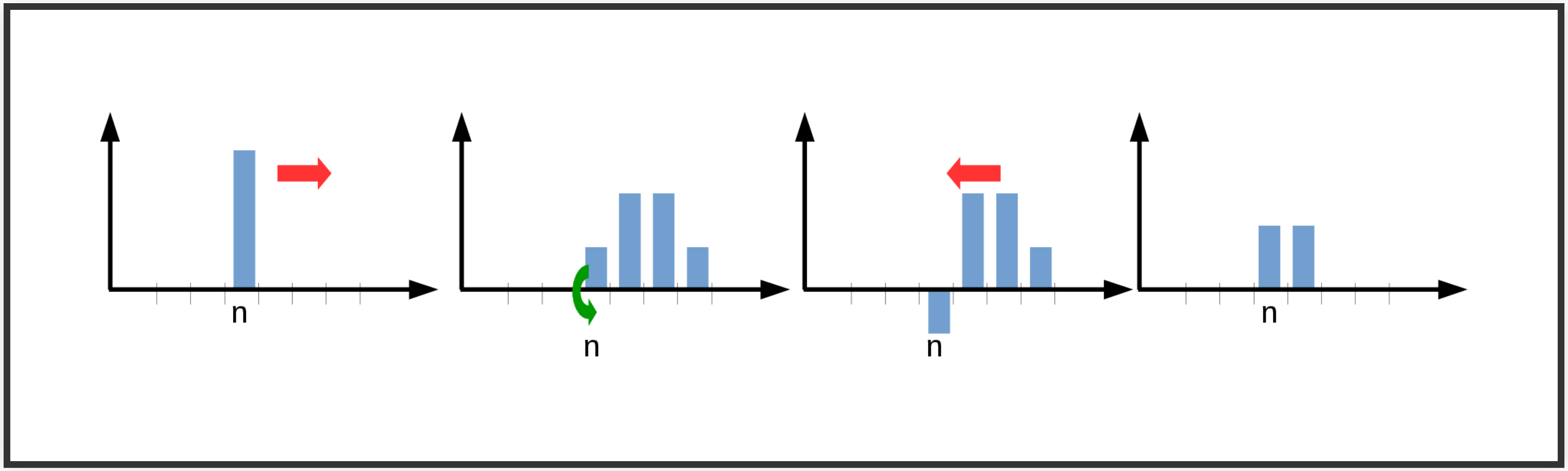
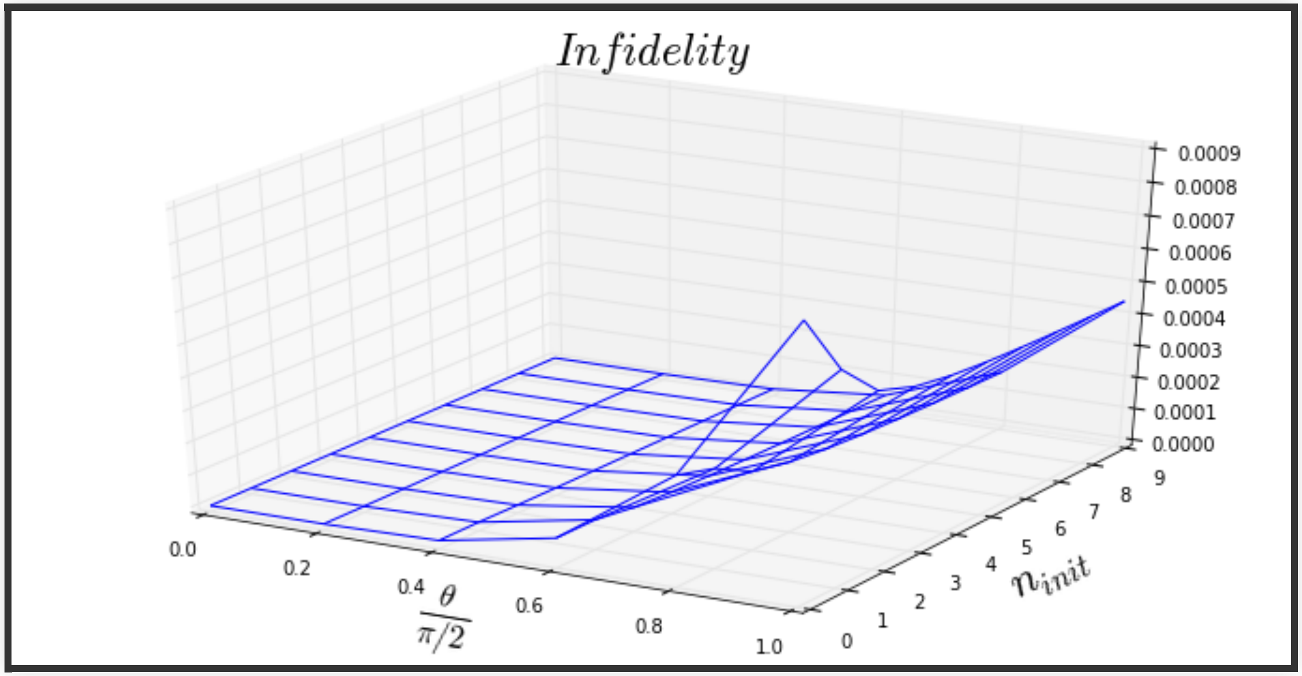


EXPLICIT ALGORITHM - N TO N+1

Optimize $F = \left| \langle \mathrm{target} | \hat{U}_n | n \rangle \right|$ wrt α_1, α_2, and α_3 with some good initial guesses.



EXPLICIT ALGORITHM - $|0\rangle$ TO $|\psi\rangle$

Restrict to "non-negative" $|\psi\rangle$

$$|\psi\rangle = |\mathbf{target}\rangle = \sum_{n=0}^{N} c_n\,|n\rangle,\;\; c_n \geq 0$$

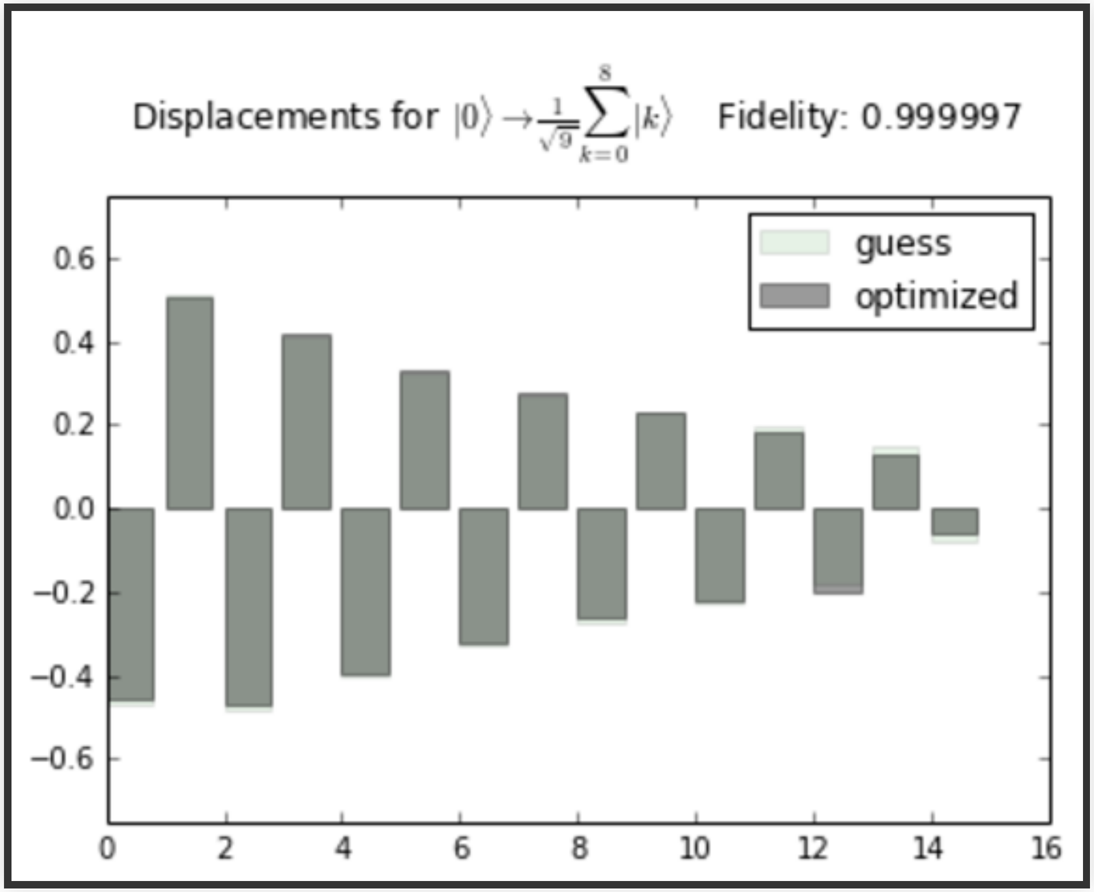
Construction by "unrolling":

- Requires N runs of the previous algorithm (n to n+1) (or just a dictionary lookup)
- Optionally a "global" optimization can be run over all parameters (after simplifications this means an optimization over $2N+1$ parameters)

Apply a final SNAP gate to impart any missing phases.

EXPLICIT ALGORITHM - $|0\rangle$ TO $|\psi\rangle$

Fidelity better than 0.999.



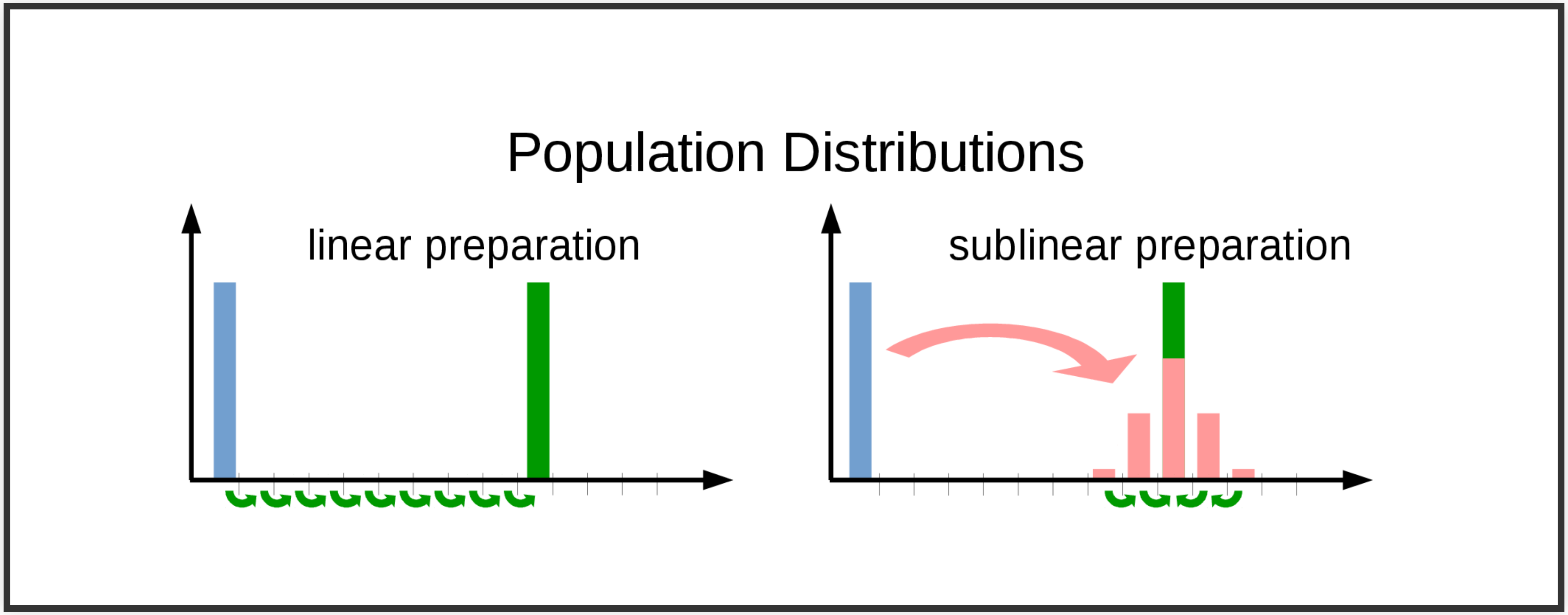
FINAL COST

To create an arbitrary N-dimensional state

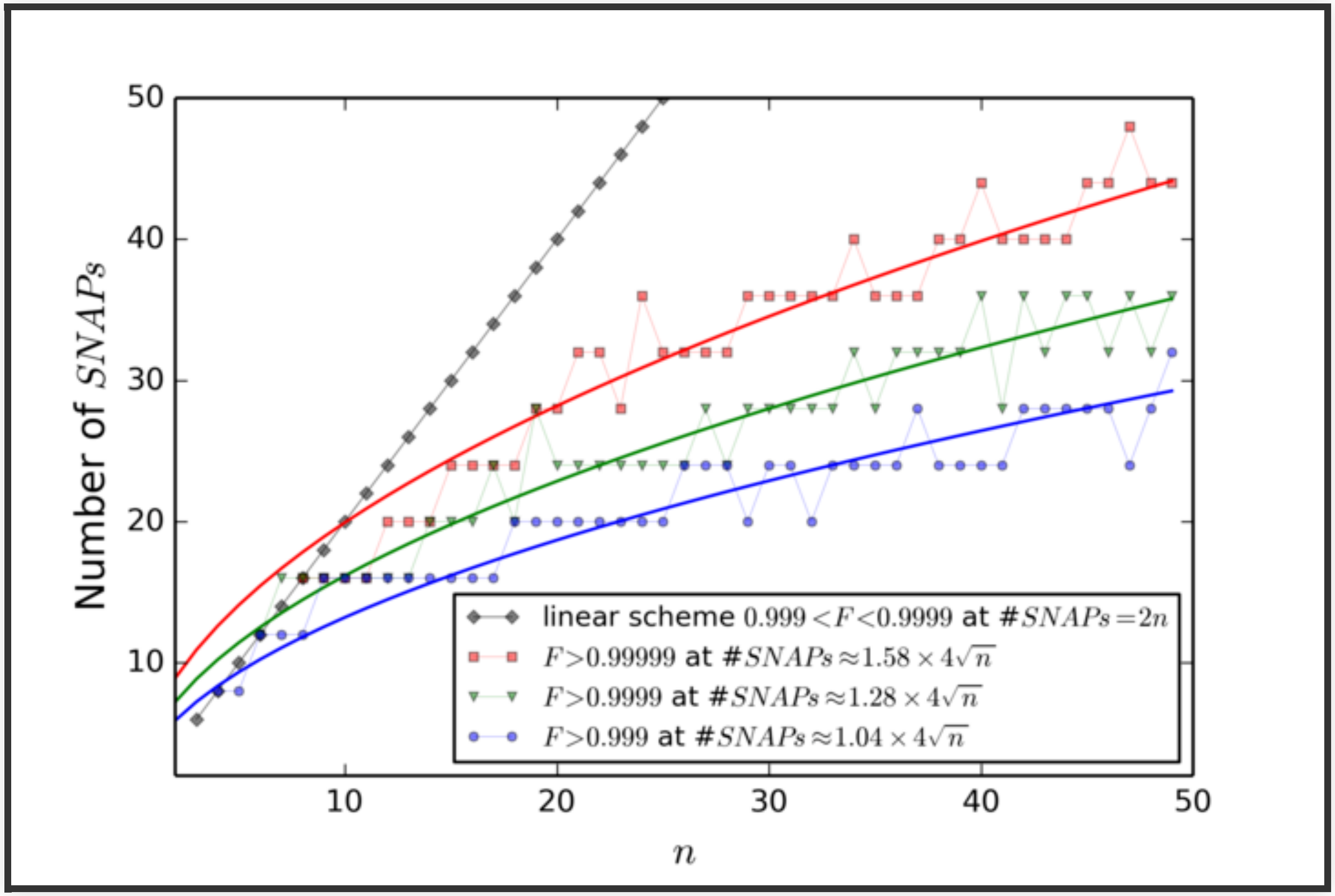
- To find the pulses
  - N optimizations over 3 parameters (or N dictionary lookups)
  - 1 optimization over 2N+1 parameters (takes ~5seconds in practice)
- To implement the pulses
  - 2N+1 displacement gates
  - 2N SNAP gates

SUPER EFFICIENT FOCK STATE PREPARATION

- Displace to coherent state $|\alpha\rangle = D(\alpha = \sqrt{n}) \mid 0\rangle$
- Use $\mathcal{O}(\sqrt{n})$ rotations to "fold" it into $|n\rangle$



SUPER EFFICIENT FOCK STATE PREPARATION



UNIVERSAL CONTROL

UNIVERSAL CONTROL

We want to perform an arbitrary unitary operation on the oscillator state (even when the state is unknown), not just prepare a target state from another given state.

- Preparing a 2x2 rotation **sub**matrix
- Chaining 2x2 **sub**matrices into an NxN unitary matrix

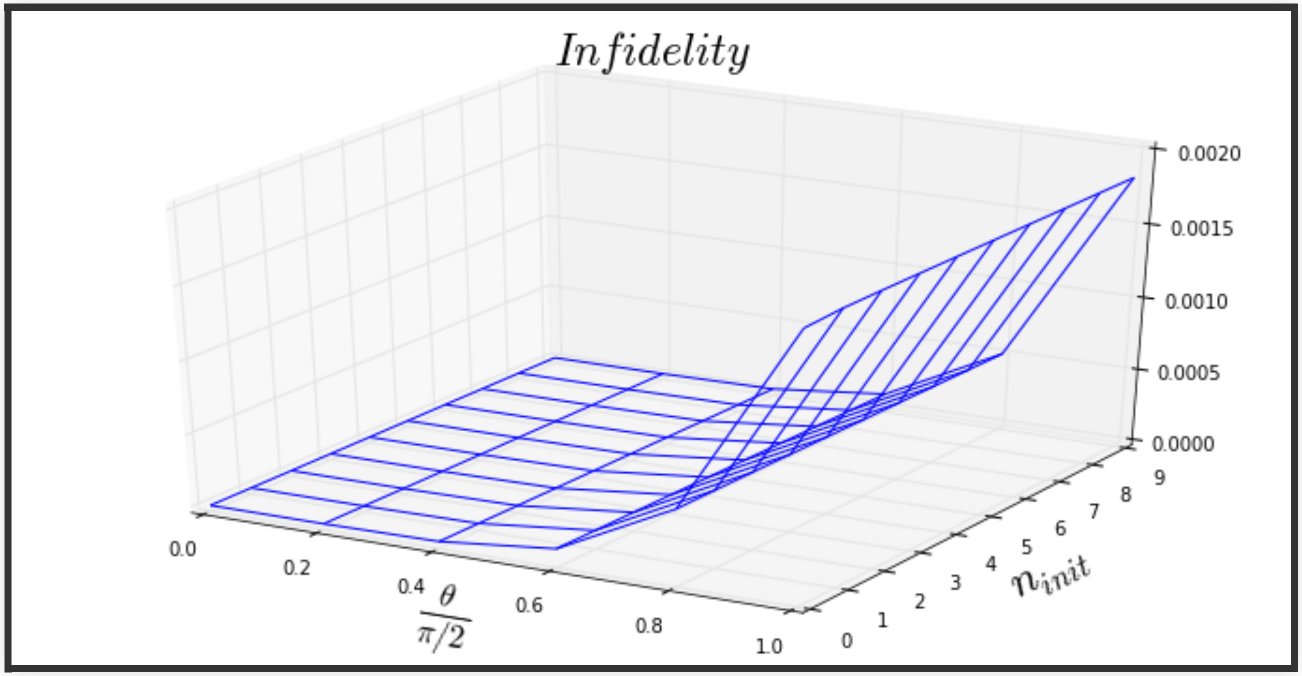
2-BY-2 ROTATION SUB-MATRIX

As in the case of state preparation for
$$|n\rangle \rightarrow \cos(\theta)\,|n\rangle + \sin(\theta)\,|n+1\rangle$$
we want to use
$$\hat{U} = \hat{D}(\alpha_1)\hat{R}_n\hat{D}(\alpha_2)\hat{R}_n\hat{D}(\alpha_3)$$
to implement
$$\hat{U}_{target} = \left[\begin{array}{c|cc|c} Id_{n\times n} & \multicolumn{2}{c|}{0} & 0 \\ \hline 0 & \cos\theta & -\sin\theta & 0 \\ & \sin\theta & \cos\theta & \\ \hline 0 & \multicolumn{2}{c|}{0} & Id \end{array}\right]$$
on the $\{|n\rangle, |n+1\rangle\}$ subspace.

2-BY-2 ROTATION SUB-MATRIX

Optimize $F = \left| \frac{1}{N_{cutoff}} Tr\left(\hat{U}^{\dagger} \hat{U}_{target}\right) \right|$ wrt α_1 and α_2

$(\alpha_3 = -\alpha_1 - \alpha_2)$



N-BY-N UNITARY MATRIX

We want to construct $\hat{U}_{target}$

$$\hat{U}_{target}^{-1} = \left[\begin{array}{c|c} \hat{W}_{n\times n} & 0 \\ \hline 0 & Id \end{array}\right]$$

N-BY-N UNITARY MATRIX - REMOVING A COLUMN

We can apply a SNAP gate to make the last column positive and then chain $N-1$ 2-by-2 rotations:

$$\hat{V}_{n-1,n} \dots \hat{V}_{1,2} \hat{S}_n \hat{U}_{target}^{-1} = \left[\begin{array}{c|c} \begin{array}{c|c} \hat{W}_{n-1\times n-1} & 0 \\ \hline 0 & 1 \end{array} & 0 \\ \hline 0 & Id \end{array}\right]$$

N-BY-N UNITARY MATRIX - REMOVING A COLUMN

Removing a column requires $N-1$ 2-by-2 matrices. To ensure high fidelity, at the end of the column removal we perform a global optimization over all $2N$ displacement parameters.

The cost function is the "leakage" outside of $\hat{W}_{n-1\times n-1}$

$$\left[\begin{array}{c|c}\begin{array}{c|c}\hat{W}_{n-1\times n-1} & 0 \\ \hline 0 & 1\end{array} & 0 \\ \hline 0 & Id\end{array}\right]$$

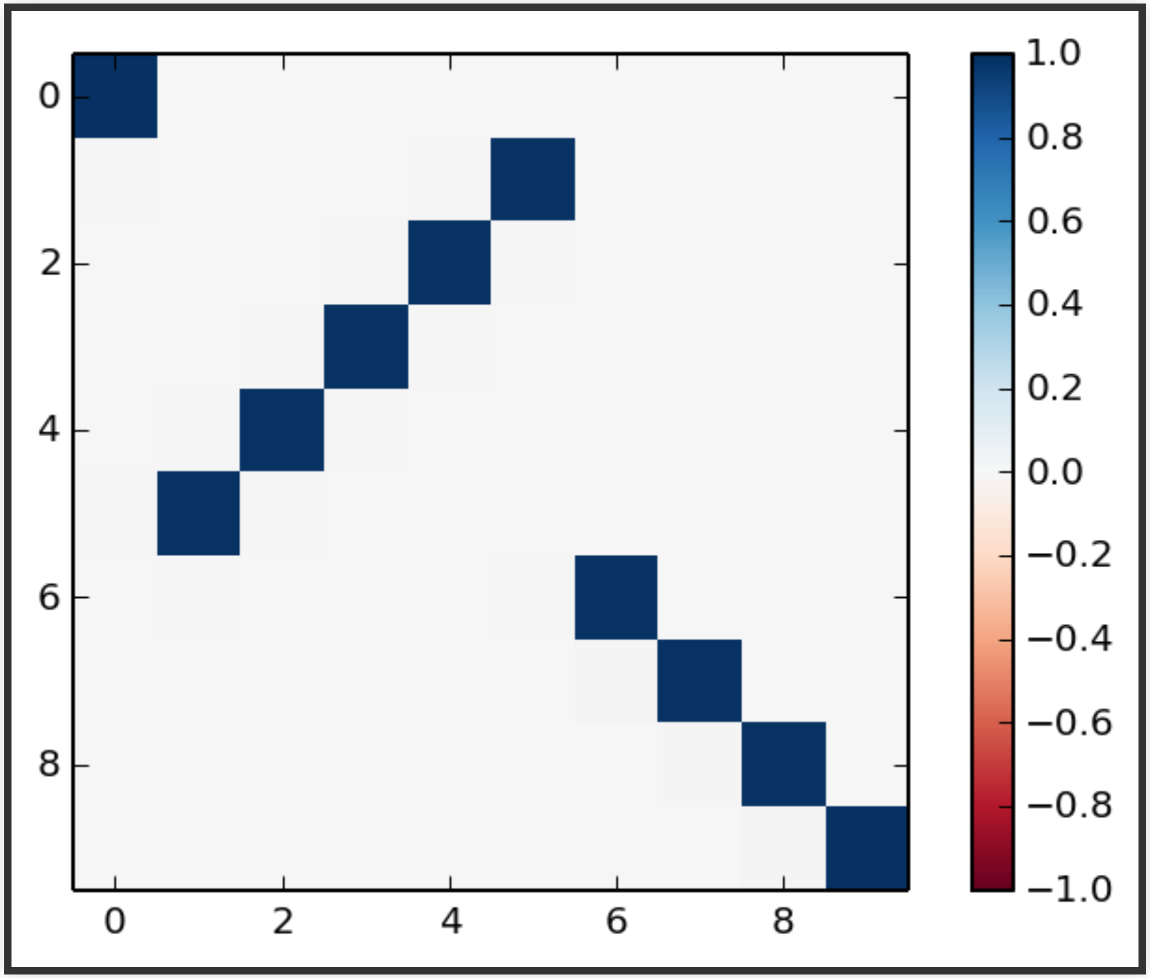
N-BY-N UNITARY MATRIX

We repeat the procedure for all columns. Then we do one final global optimization over all the displacement coefficients (~N^2 of them) against the fidelity

$$F = \left| \frac{1}{N_{cutoff}} Tr \left(\hat{U}^{\dagger} \hat{U}_{target} \right) \right|$$

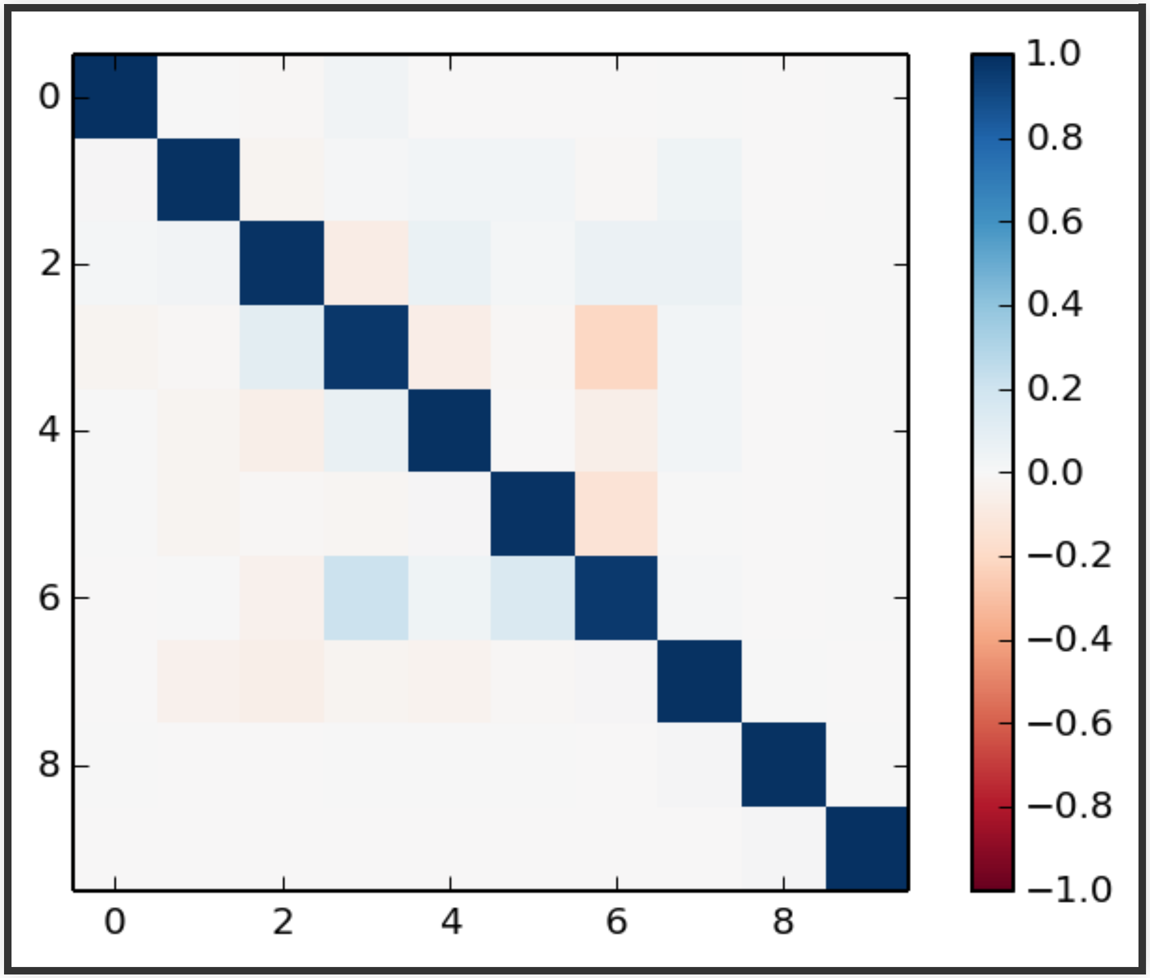
N-BY-N UNITARY MATRIX

Target: permutation matrix



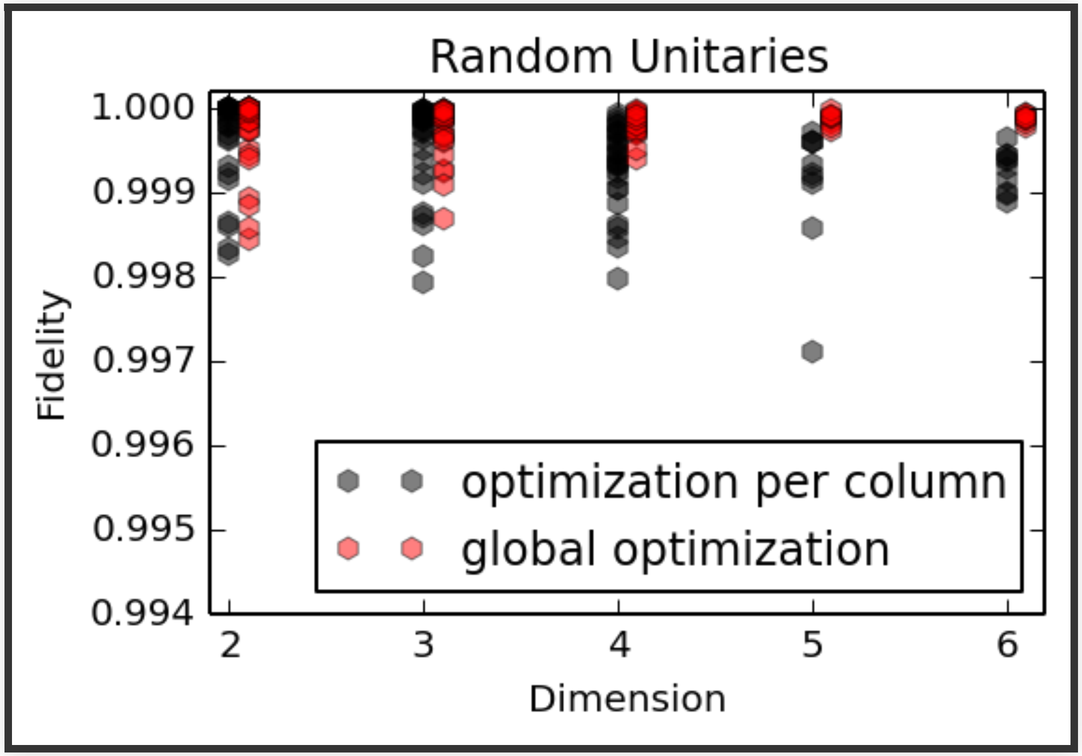
N-BY-N UNITARY MATRIX

Result: $\hat{U}_{target}\hat{U}^{-1}_{constructed}$



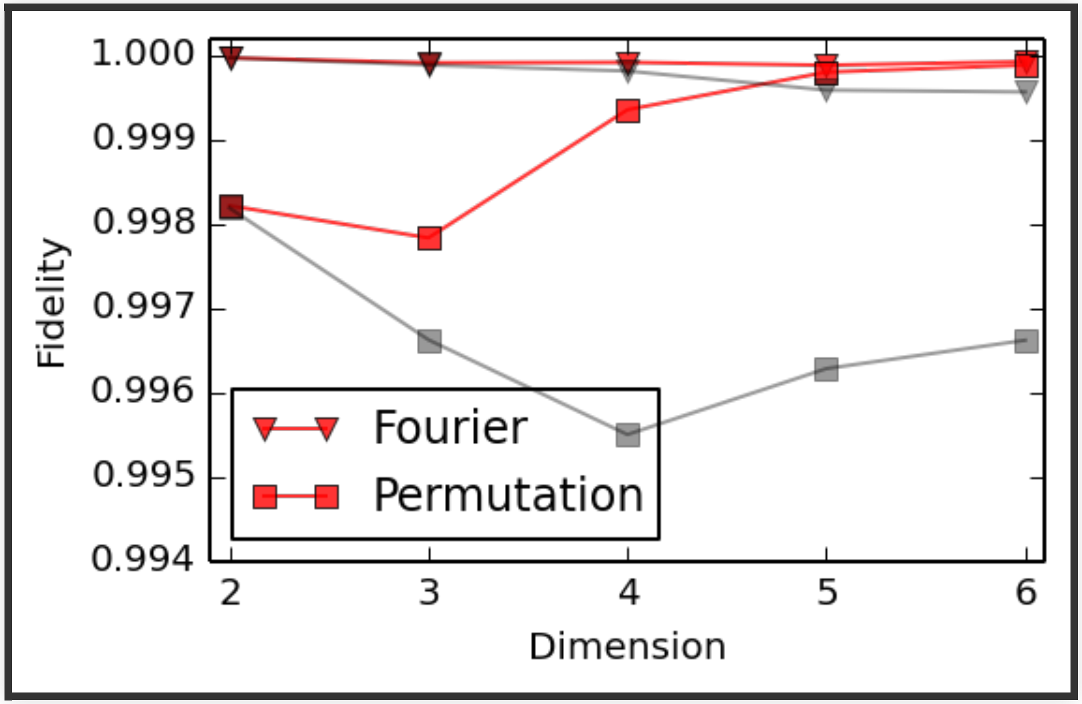
N-BY-N UNITARY MATRIX

Fidelity for random matrices:

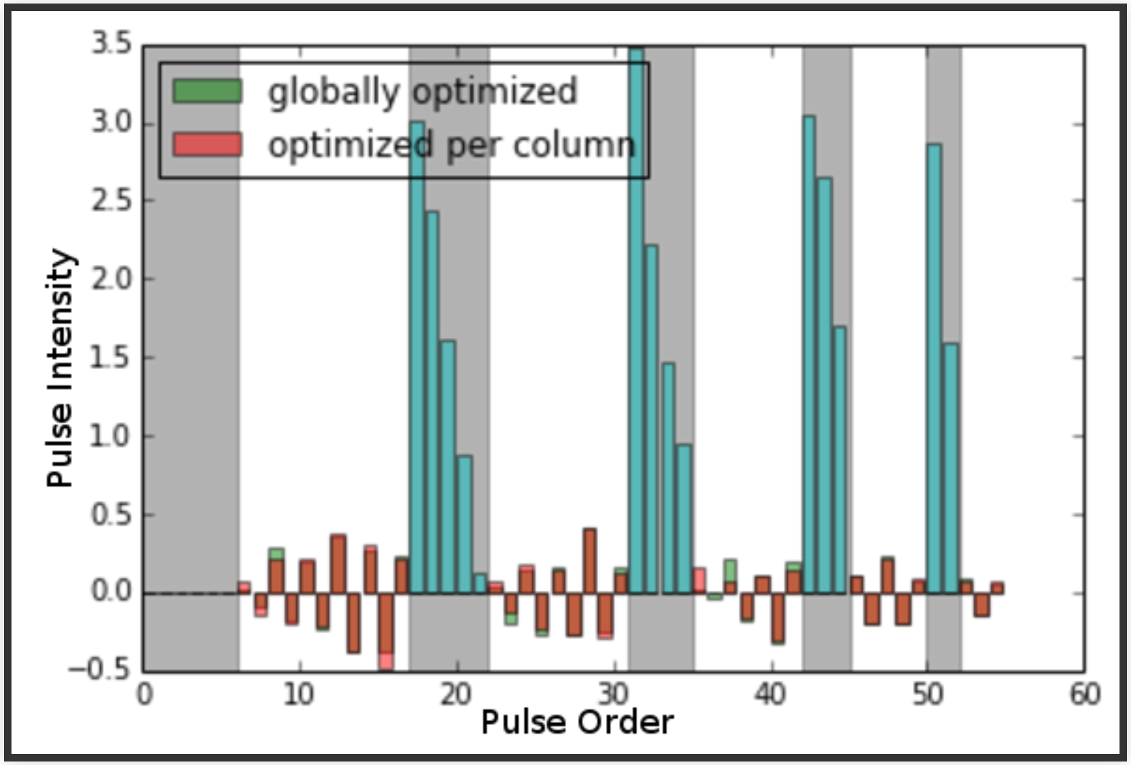


N-BY-N UNITARY MATRIX

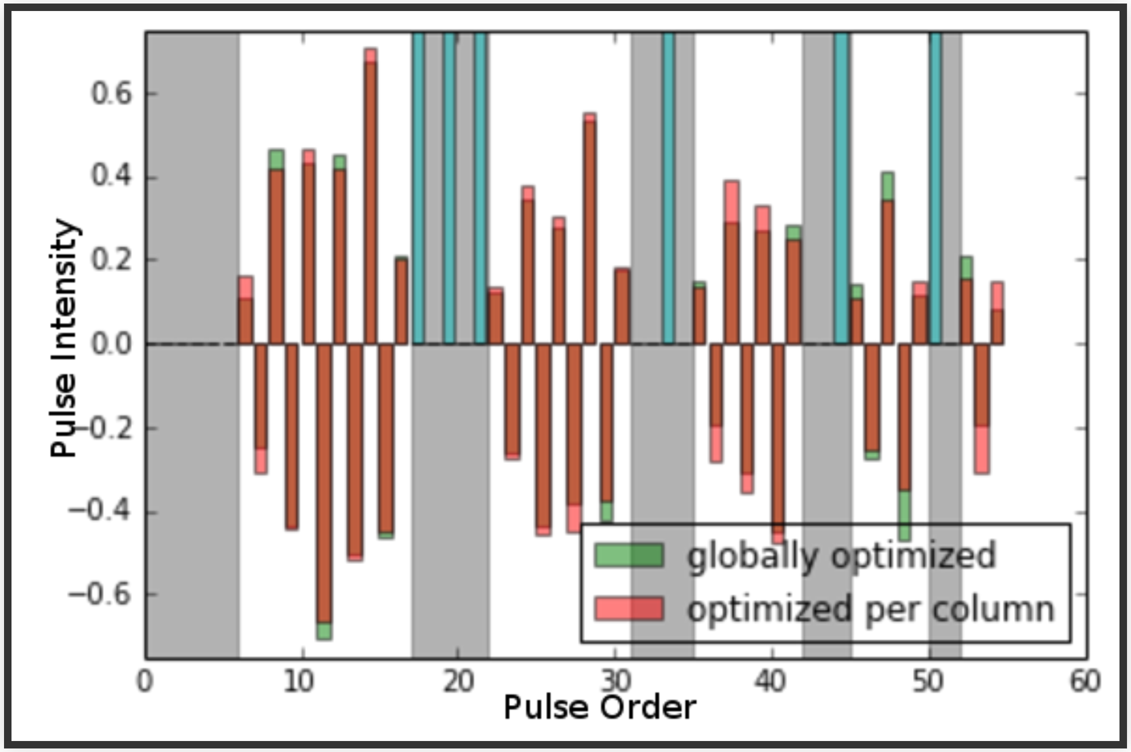
Fidelity for Fourier and Permutation matrices:



PULSES FOR 6-BY-6 FOURIER MATRIX:



PULSES FOR 6-BY-6 PERMUTATION MATRIX:



FINAL COST

To create an arbitrary N-by-N unitary matrix

- To find the pulses (takes <5 minutes in practice)
  - $\frac{(N-1)N}{2}$ optimizations over 3 parameters (or dictionary lookups)
  - $N-1$ optimization over 2N+1 parameters or less
  - 1 optimization over $\approx N^2$ parameters
- To implement the pulses
  - $\frac{(N-1)N}{2}$ rotations (each is 3 displacements and 2 SNAPs)
  - $N-1$ additional SNAPs

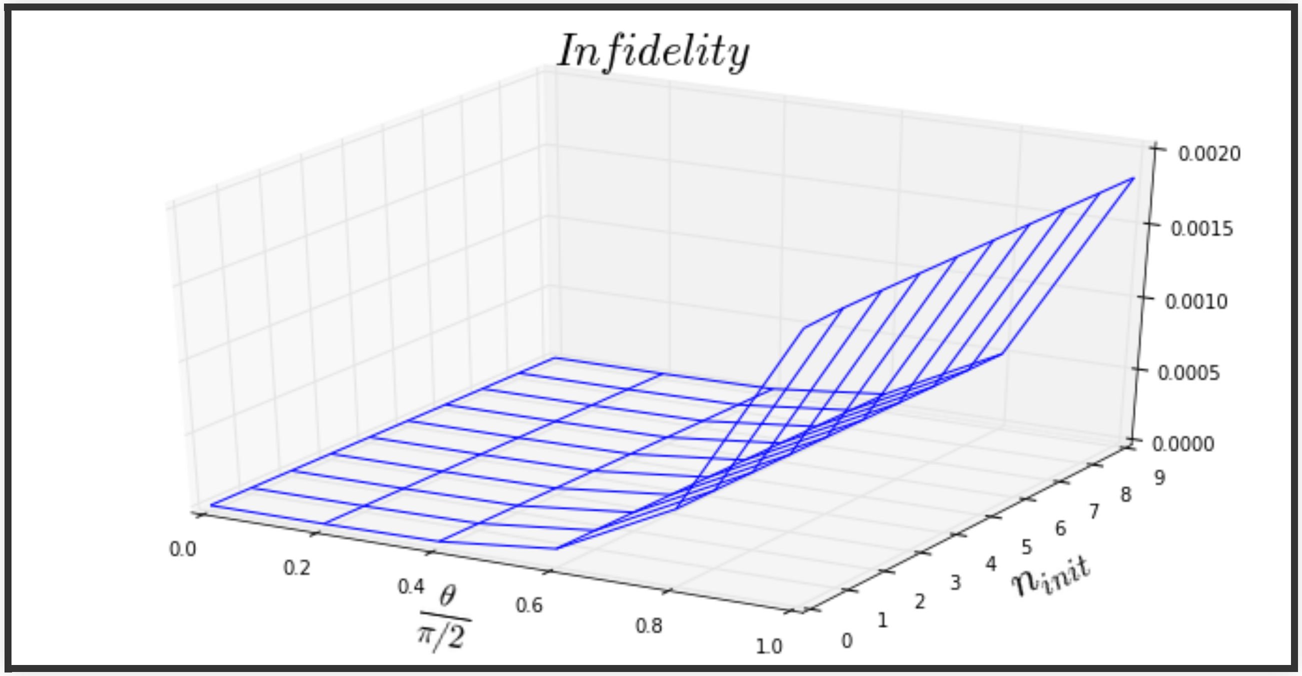
UNIVERSAL CONTROL AND UNIVERSAL COMPUTATION

- Constructing Arbitrary State
  - (efficient)
- Performing Arbitrary Unitary
  - (efficient)
- Encoding Qubits in the Hilbert Space
  - (can be exponentially expensive)

Efficient scheme to control a multi-level system that can store multiple qubits of information.

ASYMPTOTIC FIDELITY

Look at small θ:



ASYMPTOTIC FIDELITY

For certain elegant definition of infidelity:

- $\operatorname{infid}(U_{target}) = \|U_{target} - U_{realization}\|$
- where for any M, $\|M\| = \sup_{\||\psi\rangle\|_2=1} \|M|\psi\rangle\|_2$

We can prove

$$\operatorname{infid}(U_{N\times N}) \leq M\left(\frac{N}{n_{snaps\ per\ SO(2)}}\right)^2$$

ASYMPTOTIC FIDELITY

Given that $\#\text{gates} \propto n_{snaps\ per\ SO(2)} N^2$:

$$\#\text{gates} \propto \frac{N^3}{\sqrt{\text{infid}}}$$

CONCLUSION

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- Efficient State Preparation
  - Including an Optimization for Fock States
- Efficient Universal Control
  - Satisfactory at "first-pass" level
  - Remains Efficient in the Asymptotic Regime
- People are Already Implementing it on the 4th Floor

OFF-TOPIC TOPICS

SUBLINEAR PREPARATION OF 'SPARSE' STATES

The preparation of Fock states by folding can be generalized to the preparation of any "sparse" state where the population is centered around a single Fock state.

UNIVERSAL CONTROL INCLUDING THE SPIN

We were working on only half the Hilbert space (the $|g\rangle$ subspace).

If we can:

- Switch χ on and off both for the ground and for the excited state.
- Add a $\hat{\sigma}_z \mid n\rangle\langle n|$ term to the Hamiltonian on command.

We can easily extend the protocol to work on the entire Hilbert space.

EFFICIENT READOUT SCHEME

Ask Chao (MLS, Mar 30, 2015) and Reinier (MLS, next week).

Implementing a binary search for a Fock state using a SNAP-like gate and conditional measurement.

EXPERIMENTAL IMPLEMENTATION

Reinier will be presenting it next week. They are designing even shorter pulses with some in-depth numerical optimizations.

This is why experimental scientists hate theoretical scientists.

ACTUAL CONCLUSION

ACTUAL CONCLUSION

Efficient, Fast and Extendable New Protocol for Universal Control

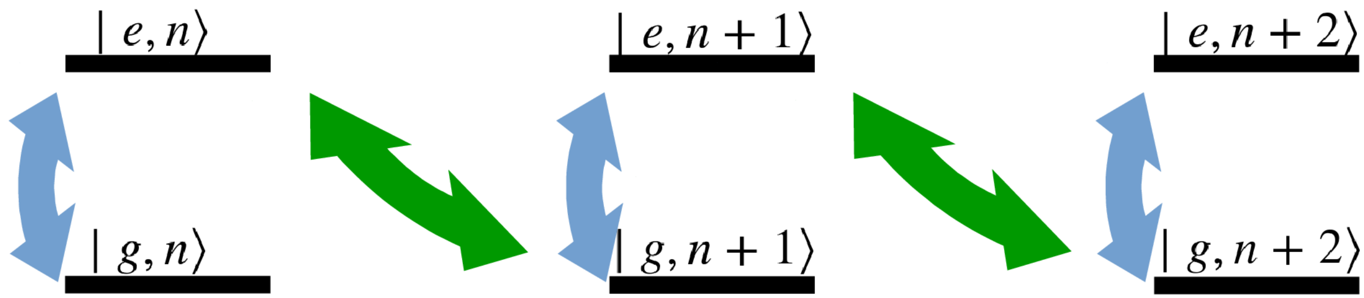
Thank you for your attention!

Questions?

(image credit to SMBC comics)

JC

STATE PREPARATION IN JC



$$H = -\frac{1}{2}\Delta\hat{\sigma}_z + H_{green} + H_{blue}$$

$$H_{green} = \frac{1}{2}g(e^{i\beta}\hat{a}^\dagger\hat{\sigma}_- + h.c.)$$

$$H_{blue} = \frac{1}{2}\chi(\cos\phi\hat{\sigma}_x + \sin\phi\hat{\sigma}_y)$$

C. K. Law and J. H. Eberly, Phys. Rev. Lett. 76, 1055 (1996)

UNIVERSAL CONTROL IN JC

Their SNAP-like gate is very expensive (they do provide faster non-analytic version).

$$T_{total} \propto \frac{N^{18.5}}{\mathrm{infid}^3}$$

Brian Mischuck and Klaus Mølmer, Phys. Rev. A 87, 022341 (2013)